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The aerodynamics of carved stone balls

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ABSTRACT

Over 400 intriguing carved stone balls have been found in Scotland, often described in museums and journal articles as 'unknown in function' or 'probably ritualistic'. This article offers a new hypothesis for their use, demonstrating that these balls are unexpectedly well optimized, in terms of their aerodynamics and mass, for unaided throwing by hand. This may explain their otherwise surprising uniformity in size and weight and their generally rough surface texture (which reduces the air drag), due to carving and/or picking-out left unpolished. The mass distribution for over 200 examples held in various Scottish museums and archive stores is presented in support of this hypothesis.

INTRODUCTION

Many Scottish museums feature showcases of intriguing carved stone balls, found essentially only in Scotland and usually attributed to Neolithic times. The explanatory annotation generally incorporates expressions such as 'hundreds found', 'enigmatic', 'perhaps used for some ritual purpose'. A number of papers have been written (Smith 1876, 29–62; Mann 1914, 407–20; Marshall 1976, 40–72; Edmonds 1992, 179–93), in which a variety of suggestions for their original function are advanced, including standard weights, mace heads, bolae, gaming pieces, divination accessories, spools, dance instructions, currency and so on. In Edinburgh today, in Festival Square on Lothian Road, there is a monumental tribute to these ancient balls, commissioned by the Edinburgh International Conference Centre (illus 1). The modern sculptor of this work, Remco de Fouw, has made full use of the range of artistic effects possible by juxtaposing balls of widely varying sizes, mostly finished with a dark, highly polished spherical surface and relatively pale machined grooves. It could perhaps be argued that if the original balls were also intended for artistic or ritual purposes, they would be found with similarly disparate dimensions and contrasted surface textures. Instead, however, John Smith was 'struck first by their great general resemblance in size and weight' (Smith 1876, 29–62), while the surface finish is in all but a few cases noticeably rough, with the fabrication marks not polished out.

Many people are aware that the dimpling on projectiles like golf balls somehow extends their driving range compared to that of a shiny-smooth sphere. Empirical optimization of the dimpling pattern through the 19th and early 20th century was eventually backed by experimental research and later still by theoretical modelling. In the 1970s experimental work (Achenbach 1974, 113–25) produced curves of air drag versus Reynolds number (proportional to speed, see later sections) for varying roughness depth. These showed that the critical speed for transition to a low drag coefficient (and hence a significantly increased range) reduced strongly as the roughness or dimple depth was increased, an observation which brings the Scottish carved stone balls to mind. The knobbliness and ubiquitous roughness of these balls gives rise

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ILLUS 1 Festival Square, Edinburgh 2004

to a critical velocity for low drag that would be readily achievable when directly thrown by hand. The aerodynamic features alone do not explain the near-constant size of the hundreds of such balls that have been found in Scotland, however.

This work reports experiments using model carved balls to verify the aerodynamic characteristics and, based on these, summarizes ballistic analyses within the constraints of a hand-thrown stone ball, including back-spin (the Magnus effect) and an estimate of the maximum speed a human can impart to a missile, which is a function of the weight of that missile. A simplified fitting expression allows the calculated range to be parameterized in an equation, facilitating further analysis demonstrating that there is an optimum ball diameter to achieve the maximum thrown distance.

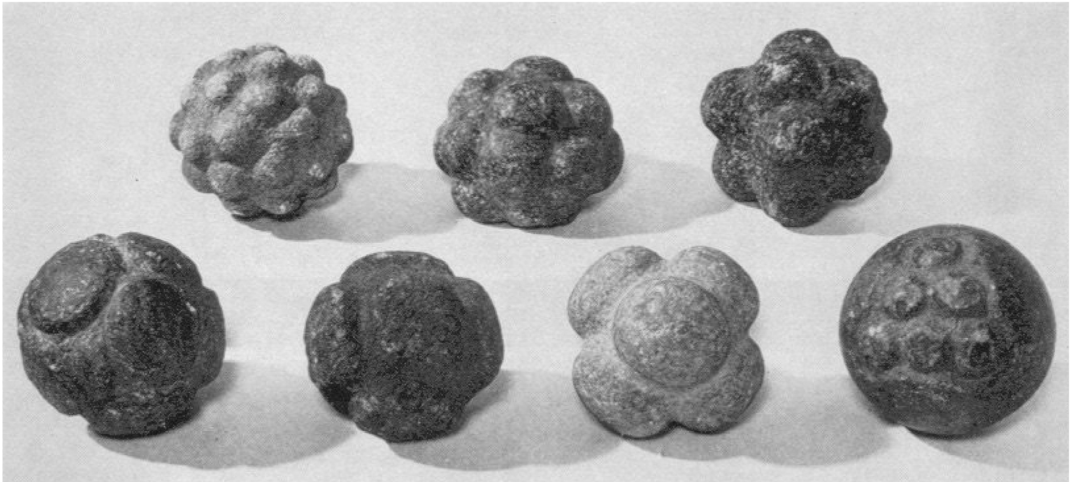
CHARACTERISTIC FEATURES OF THE SCOTTISH BALLS

DIMENSIONS AND APPEARANCE

The carved stone balls are made from various different types of stone, with densities ranging from that of sandstone (2.1g/cc) to granite (2.7g/cc). There are very many different classes, as grouped by Marshall (1976, 40–72). Some are very shallowly carved while others are deeply incised to create gross knobbles, a good fraction of the total radius in depth. Practically all seem to be deliberately left very rough, either from the natural grittiness of the stone or from an absence of grinding or polishing of the innumerable small craters left by the shaping process (picking out). Typically the roughness has characteristic dimensions of about 1–2mm. At some critical speed in moving through the air, as will be shown below, the roughness and/or the carvings will promote fine-scale turbulence and thus inhibit the creation of a large turbulent wake, the main source of drag at high speed.

Illus 2 depicts a selection of the more common types of carved stone balls, emphasizing the deeply carved grooves, which will have taken significant effort to achieve by hand without hardened machine tools.

A study undertaken for the National Museums of Scotland in Edinburgh found that it took about 11 hours to make one four-lobe ball of this size, using stone tools and starting from



ILLUS 2 Plate 4a from Marshall (1976). Reproduced with kind permission from the Society of Antiquaries of Scotland

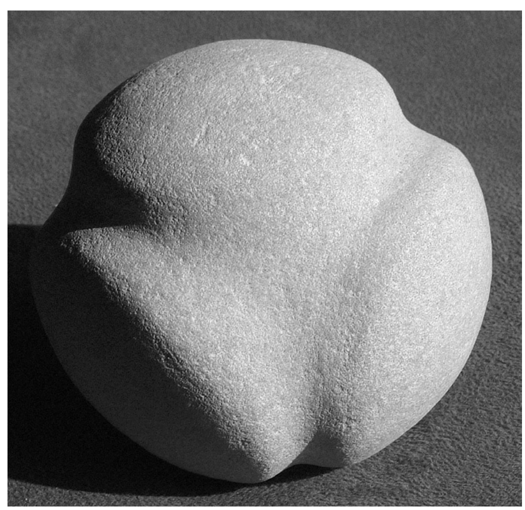
a pebble somewhat too prolate but otherwise needing little dressing to form the initial sphere (Sheridan 2005). I have obtained a very similar result, making the 540g, 81mm diameter four-lobe sandstone ball shown in illus 3 in 12 hours using only stone tools. These comprised three much harder stones shaped like cheese wedges and one smoothly rounded stone of somewhat harder sandstone (all five picked up during a short walk on Golspie beach). The original pebble was again essentially the desired spherical shape and size, apart from being slightly prolate. No separate hammer was employed, just the work-piece in one hand and the selected stone tool in the other, using a steady light chipping and, to finish, a grinding action.

Clearly, balls like the superbly decorated Towie ball (Clarke 1985) will have taken very much longer to polish and then carve with delicate patterns; however, only a very small percentage of the overall collection is carved with such fineness (Marshall 1976, 40–72).

Marshall observed that:

> Of the 387 balls on my cards I have handled all but about a score. In every case the ball gives the feeling of having been much handled. This is more than the smooth finish of a well-made object. Each and every one is a craftsman's job; many are real works of art' (Marshall, 1976, 40–72).

She also noted in that article that 'Three hundred and seventy-five of these [387 at that time] balls are much the same size, with a diameter of about 70mm'. I would add the observation (from



ILLUS 3 Four-lobe sandstone ball made by the author, solely with stone tools

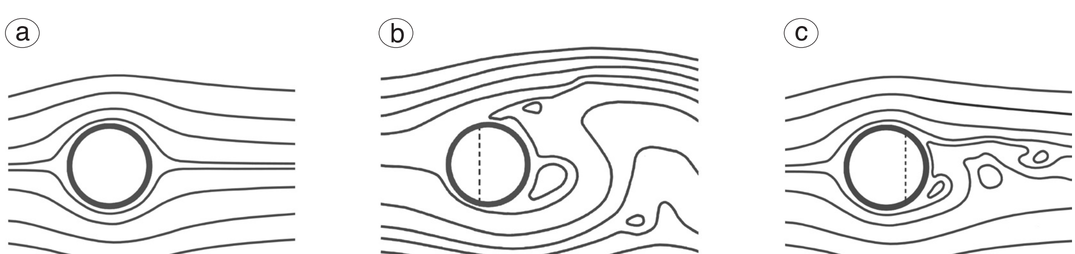
handling over 200 examples) that most are quite rough, ie apparently unpolished, generally left with the picking-out scars. The majority depart considerably from the symmetry of 'Platonic solids', in many cases apparently demonstrating an ad hoc work process, or best-endeavours recovery from fabrication problems such as insufficient remaining space on the surface or unintended fractures in the features being formed.

One can ask, 'Why are they essentially all the same size? Why not considerably smaller or larger, like the modern examples in Festival Square?' In the following sections it will be shown that this may be due to an empirical optimization process, such that the size and form is well suited to moving through the air at about 25–35m/s (or up to 75mph), the maximum speed a human could possibly throw objects of this weight.

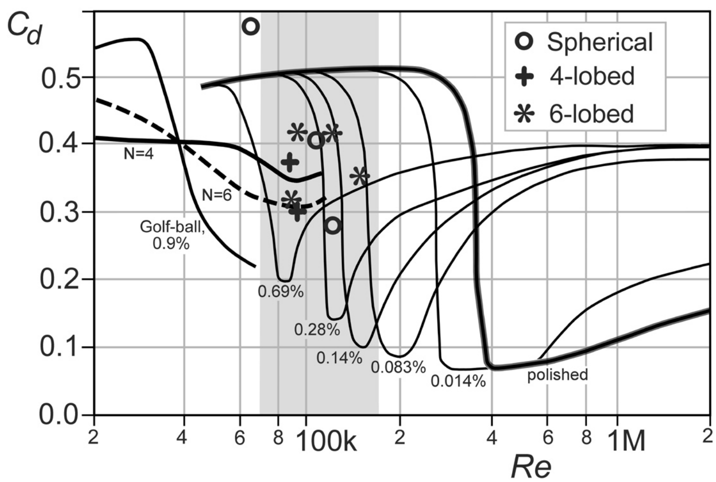
AERODYNAMIC DRAG

The theory of the 'boundary layer', meaning the transitional zone between the parts of a fluid (air for the suggested original application and water for some of the tests) touching and therefore moving with the surface of a moving object and the main bulk of the fluid far from that surface, explains the aerodynamic behaviour of strongly roughened balls such as these (Schlichting 1979). A dramatic phenomenon in the movement of blunt objects through fluids is a transition at some speed to a finely turbulent boundary layer. Above this critical speed there is a range of speed in which the gross fluid behaviour mimics that of streamlined flow (ie closing in smoothly behind the object instead of stirring up a large wake), reducing the drag forces considerably. This behaviour is sketched in illus 4, showing firstly streamlined flow (very low speed), then a flow with separation of the boundary layer from the sphere somewhat ahead of the mid-point of the sphere (ie at intermediate speed) and, finally, the flow with a 're-attached' boundary layer, now detaching significantly behind the mid-point (ie at high speed).

The important feature is the width of the wake, clearly very much smaller in the re-attached case than the case with detachment ahead of the mid-point. Achenbach (1974, 113–25) studied this phenomenon in detail for polished and lightly roughened spheres, with the results shown as the set of curves extending towards the right (illus 5). The x-axis here is Reynolds number, ie velocity times diameter, divided by kinematic viscosity ($\approx 1.5 \times 10^{-5}$ m^2/s for air). Reynolds number is chosen rather than speed because one of the properties of fluid dynamics is that the measured flow patterns, drag coefficients etc are the same when the Reynolds number (which is effectively the ratio of inertial to viscous forces) is the same. Thus valid tests can be carried out in water as well



ILLUS 4 Examples of flow past a sphere (the flow being from left to right). (a) Streamlined flow; (b) flow with boundary layer detachment ahead of the mid-point of the sphere (where shown by the dotted line); (c) flow with a re-attached boundary layer, detaching where indicated by the dotted line

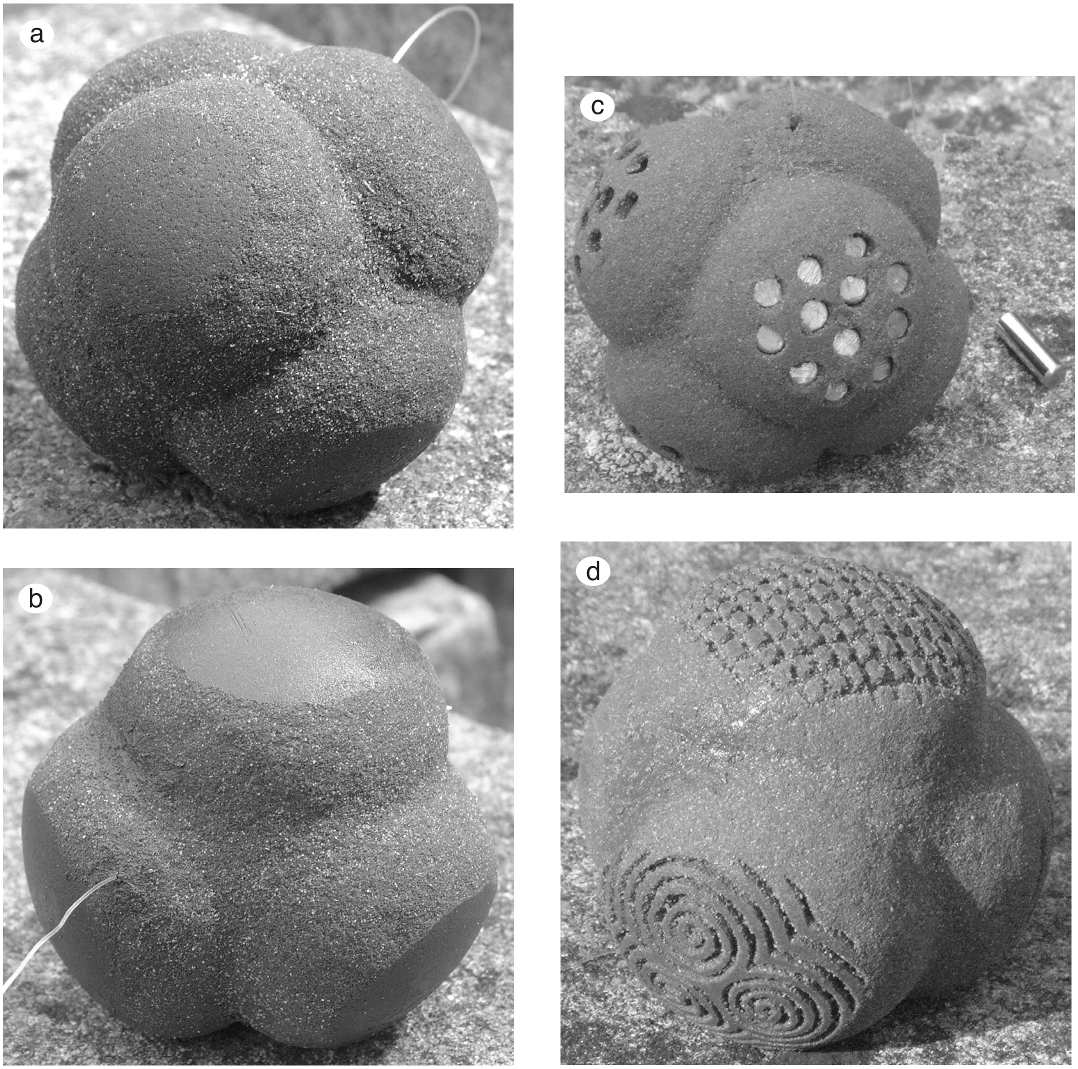


Illus 5 Variation of drag coefficient C_d with Reynolds number Re for roughened balls. The parameter given as a percentage against each of the curves is the effective size of the roughness elements for the roughened sphere under test, divided by its diameter. The model balls described in the text are represented by the loci annotated N = 4 and N = 6 for air jet tests, and the indicated points for water tests: circles are for the spherical ball, the + sign is for four-lobed and the asterix for six-lobed balls. The grey area represents the speed range of interest, corresponding to speeds of 14–38m/s for 70mm diameter balls moving in air

as air, but the speed has to be 15 times smaller to keep the Reynolds number the same. The y-axis is drag coefficient, $C_d = F_{drag}/(\rho A v^2/2)$ where F_{drag} is the drag force, ρ is the density of the air, A the frontal area of the ball and v the relative air velocity. Inspection of the very bold curve labelled 'polished' in illus 5 reveals the basic effect of drag coefficient reducing above a critical speed. The other curves show that increasing the surface roughness (which seeds the fine-scale turbulence that delays detachment of the boundary layer) causes the drag coefficient to drop at lower speeds, but that it does not reduce to such low values as with fine-scale features.

This trend has more recently been confirmed by Aoki and Nakayama using balls with parallel grooves ranging up to about 1.6% of the ball diameter in depth (Aoki 2000). Tests have also been undertaken by the author, using model balls described below, to verify that the shapes represented by the Scottish carved stone balls also follow this tendency, in particular that a gross surface deformation of several per cent of the diameter does not create any adverse effect (like an 'air brake' for instance) but simply holds the minimum drag coefficient to around 0.3. The drag coefficients of the model balls were determined both in free-fall in water where the ambient turbulence was very small and in an air jet, in which the average velocity turbulence was approximately 1–2%. Turbulence is much weaker in effect than surface roughness (when both are stated as percentages) but the effect is broadly similar and at this level only the rougher balls could meaningfully be tested in the air jet, so the spherical ball used for reference is represented in illus 5 only by three points from its water test results. These suffice to demonstrate the sharp drop in C_d at Re ≈100,000 expected for a ball with roughness ≈0.3% of its diameter. The model carved balls behave essentially as expected, exhibiting fairly low drag throughout the relevant range of speed (ie Reynolds number), highlighted in grey in illus 5. The behaviour of a modern golf ball was also determined and is shown for comparison, suggesting that knobbles or fabrication roughness of around 1% of the diameter, as found in many of the Scottish carved stone balls, would also work well.

The four-knobble ball was tested first without and then with fine patterns on the four lobes, but this had no discernible effect on the behaviour of its drag coefficient. The error bars on all the results shown in illus 5 obtained for this article – omitted for clarity



ILLUS 6 Examples of model balls made from loaded silicone rubber. (a) Six-lobe silicone rubber ball; (b) four-lobe silicone rubber ball; (c) adding extra mass to the six-lobe ball; (d) four-lobe ball after pattern carving

but typically ± 0.05 in C_d and a few thousand in Re – arise from fluctuations associated with the balls changing in orientation and suffering cyclic variations in the flow pattern (ie vortex shedding), together with residual experimental uncertainty in measurements of the air speed or the speed of the balls falling through the water. A final important observation from illus 5 is that very smooth balls would not fall to low drag below Reynolds numbers of about 300,000, ie speeds of over 60m/s for a diameter of 70mm, too fast for direct throwing by hand.

The model balls (illus 6) were carved from silicone rubber mixed with sand and loaded with pieces of steel. Each ball was moulded as a sphere and then marked out and carved as desired. One ball was left spherical as a reference and features surface irregularities dominated

Table 1
Characteristics of the model balls

Identifier	*Diameter (mm)*	*Knobbles*	*k/D ratio (%)*	*Area (cm^2)*	*Volume (cc)*	*Mass (g)*
Spherical	71	0	≈0.3	39.6	187	390
N=6a	70	6 smooth	7.0	36.4	165	295
N=6b	70	6 smooth	7.0	38.0	175	520
N=4a	70	4 smooth	7.7	37.2	170	316
N=4b	70	4 carved	7.7	37.2	168	311

by the texture created by the adjacent sand grains. Because a fraction of each sand grain is hidden in the silicone rubber, the effective roughness is somewhat less than the mean sand grain diameter of 0.24mm. The principle characteristics of these balls are summarized in Table 1, including the frontal or cross-sectional area used in the calculation of drag coefficient from the observed drag force. In Table 1, k/D is the ratio of the size of the roughness element to the ball diameter.

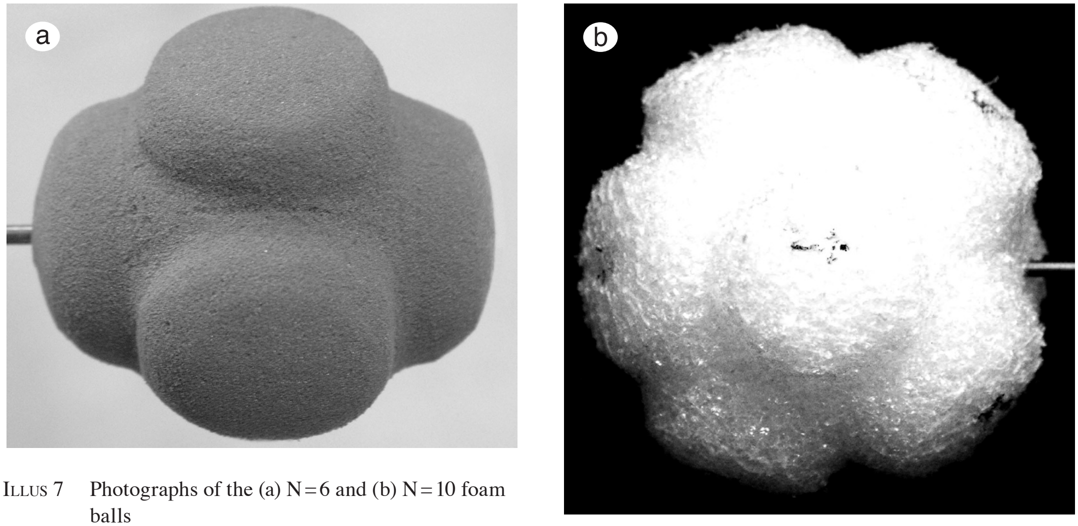
AERODYNAMIC SPIN EFFECT

The established literature on sports balls features some indications that increasing the surface deformation (eg the number of seams in baseballs) raises their lift coefficient (ie Magnus effect) for a given value of 'spin parameter' (ie the ratio of back-spin tip-speed to velocity through the air). A review of data for sports balls was produced by Alaways & Hubbard (2001, 349–58), who added some data of their own for baseballs. The back-spin behaviour of a second set of model Scottish balls was determined by the author, with a finely textured 'smooth' sphere and a modern golf ball (with a pseudo-random pattern of dimples of ≈2.5−6mm diameter) for comparison. In this case, foam balls were used which were easily carved and naturally featured some textural roughness (and were light enough to avoid whirling instabilities when spun on a thin rod; Table 2 and illus 7).

These balls were mounted on a 2.5mm diameter rod and spun by an electric motor in a 100mm diameter air jet with mean velocity up to 23m/s (Reynolds number ≈78,000 for 51mm ball diameter) and velocity fluctuations of ≈1–2% rms of the mean velocity. Illus 8 presents the data obtained using these foam balls, together with the curve for a modern golf ball tested in the same rig. In this illus, the lift coefficient is defined as $C_L = F_{lift}/(\rho A v^2/2)$ where ρ is the density of the air, A the frontal area of the ball and v the relative air velocity, while the spin parameter S is the rotational tip speed divided

Table 2
Characteristics of the Magnus effect test balls

Identifier	*Diameter (mm)*	*Knobbles*	*k/D ratio (%)*	*Roughness*	*Area (cm^2)*
Smooth	51	0	0.3	0.16mm	19.8
N=6	51	6	7.8	0.16mm	18.5
N=10	54	10	9.3	1.6mm	22.6
Golf ball	43	Hundreds	0.9	Polished	14.5



ILLUS 7 Photographs of the (a) N = 6 and (b) N = 10 foam balls

by the relative air velocity. The experimental errors were about ±0.02 in both C_L and S.

The observed lift coefficients of the golf ball and the 'smooth' ball essentially cover the spread seen by other authors and summarized by Alaways & Hubbard (2001, 349–58). The line called 'Theory' in illus 8 is a traditional analysis result strictly only valid at negligible Reynolds number (ie very low air speed), but often used as a simple expression for the Magnus effect when experimental data are unavailable or when an analysis of limited accuracy is sufficient. In the plotted coordinates, it is simply equivalent to $C_{lift} = 1.0S$.

The basic conclusion is that in the region of interest, the Magnus or back-spin effect for [models of] the Scottish carved stone balls is much the same as for golf balls and is apparently not increased by introducing the very large surface deformations represented by the knobbles.

CALCULATED THROWING RANGE

Following these experimental verifications, a large number of calculations has been made in support of this work, numerically evaluating the trajectories of such balls, with and without back-spin to make the ball carry further. The back-spin rotation speed was made proportional to the horizontal velocity to allow a plausible, progressive slowing of the rotation due to air drag. In each case the launch angle was adjusted to achieve the greatest range, being significantly less than the usual 45° when the back-spin was relatively high. To simplify the subsequent analysis in this paper, the numerical results for the optimum range have been fitted to a one-line equation for the throwing range normalized to the optimum vacuum ballistic range $R_{vac} = v_0^2/g$ (that is, launch velocity squared divided by the acceleration due to gravity). Readers not desiring to calculate the range of spheroidal projectiles launched in air should skip to the next section, perhaps while noting that subsequent analysis in this paper made use of a fitted equation, rather than calculating the throwing range point by point along the trajectory for every different type of ball. This equation is here presented in terms of the characteristic parameter 'weight divided by initial drag force', simplified to $\rho_{ave} r / C_d v_0^2$ (ie the average density of the ball times its radius, divided by drag coefficient times launch velocity squared, all in SI units) and another characteristic variable S/C_d, essentially the ratio of the lift and drag forces:

$$R/R_{vac} = 0.5[1 + \tanh\{1.1(\log_{10}(\rho_{ave}r/C_d v_0^{\,2}) + 1.5 + 0.18S/C_d)\}]$$

where $\tanh(p) = (e^p - e^{-p})/(e^p + e^{-p})$, the hyperbolic tangent of any parameter p, and S is the spin parameter defined above (but using the horizontal, not total, velocity so that it monotonically slows during the flight), included in these calculations in the simplified manner described as 'theory' in illus 8. Here ρ_{ave}, the average density, is the density of the chosen type of stone reduced by a factor representing the loss of material due to the carvings cut into the full sphere of radius r. The accuracy of this expression in matching the numerically evaluated throwing range is in the order of a few percent for the cases of interest here, with moderate drag and spin factor $S \leq 0.2$.

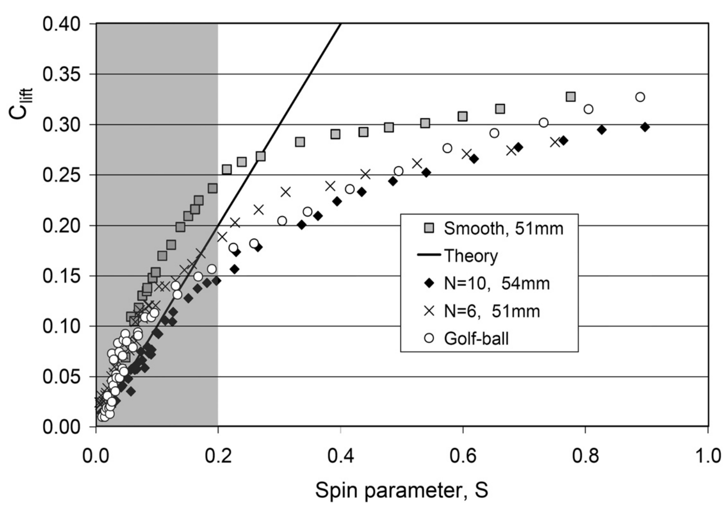
BALL SIZE AND MASS OPTIMIZED FOR THROWING

THEORETICAL OPTIMUM

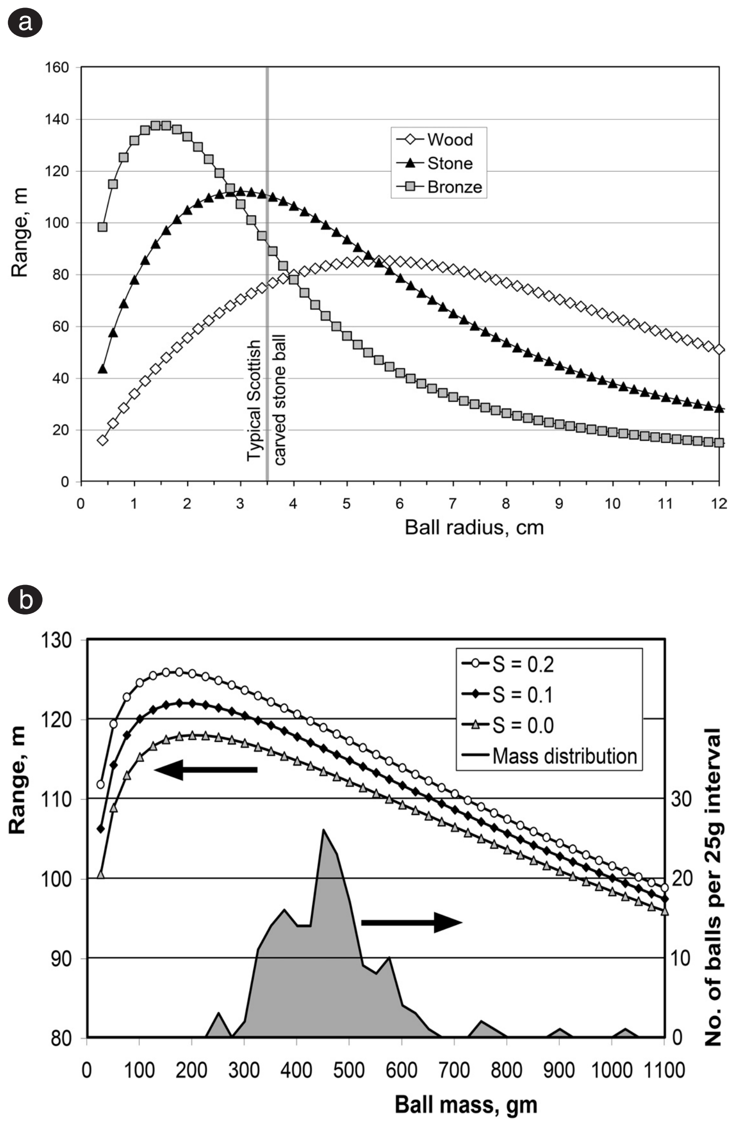
When a human throws an object, they are applying an accelerative torque to their upper body and arm, reacted by the angular inertia of their body, arm, hand and the projectile. A very simple approximation for the maximum possible throwing speed can thus be deduced, which (fitting to the known launch speeds of sports balls and Olympic throwing implements) can be taken as $v_0 \approx 7 + 40/\sqrt{(1.3 + m_{ball})}$ with units m/s and kg. (Here the '7' accounts for a sprinting speed added to the throwing speed, corrected for launching at 45° which is close to optimum for the cases of interest.) The throwing method in mind is essentially that of throwing a javelin but, because it is a ball, back-spin can be applied. The optimum shape, size and mass for a hand-thrown ball made out of stone can now be elucidated. Illus 9a–b shows the throwing range achievable for a selection of materials and back-spins, with the tip radius of the ball as the x-axis variable (illus 9a) and the mass of the ball as this variable (illus 9b). It is apparent that parameter choices that make the balls more 'draggy', ie of smaller ρ_{ave}/C_d, push the optimum tip radius, and therefore the mass, to larger values, while higher back-spin weakly favours smaller radii.

It is clear that in all these cases there is a broad optimum between 'throwing a ping-pong ball' (small radius) and 'putting the shot' (large radius) behaviour. Illus 9a shows that higher density materials (ie stone, for Neolithic peoples) are best, and illus 9b that reducing the drag coefficient and adding some back-spin offers an improvement of up to about 20% over the 'stone' case shown in illus 9a.

Illus 9b also shows that the optimum range for balls made out of stone, with drag-reducing roughness or carvings and moderate back-



Illus 8 New results of back-spin lift measurements for a modern golf ball and for foam balls: a sphere and two models of Scottish carved stone balls. The grey region is that of interest for hand-thrown balls



ILLUS 9 (a) Range optimization for hand-thrown simple spherical balls of various densities, no back-spin, drag coefficient = 0.5 (as would be exhibited by very smooth spheres). Open diamonds = wood (0.6g/cc), solid triangles = stone (2.1g/cc), grey squares = bronze (8.4g/cc). (b) Range optimization for hand-thrown sandstone balls (2.1g/cc) with knobble depth 10% of overall diameter, C_d = 0.3 and back-spin parameter (here referred to the horizontal velocity) S = 0 (grey triangles), 0.1 (solid diamonds), 0.2 (open circles). The curve blocked out in grey represents the mass distribution of the actual Scottish carved stone balls held by the Edinburgh, Inverness, Aberdeen Museum Service, Hunterian, Aberdeenshire Heritage and the Marischal College Museums

spin (rotational tip-speed up to one-fifth of the horizontal velocity), is achieved to within about 10% for any ball of mass 100–700g. This mass range brackets the actual mass distribution of the 200 or so Scottish carved stone balls that formed the basis of this study, shown as the grey-shaded curve at the bottom of illus 9b.

MASS DISTRIBUTION OF THE SCOTTISH BALLS

The inset curve in illus 9b represents the mass distribution of the balls held by the Edinburgh, Inverness, Aberdeen Museum Service, Aberdeenshire Heritage, Marischal College Museums and the Hunterian. For each point, the y-axis (on the right) represents the number of balls with mass lying within ±12.5g of the value indicated by the x-axis. This graph excludes one unusually large ball of 1422g, but this was included in the dataset statistics to yield an overall average weight of 459g, with 10-, 50- and 90-percentiles at 337g, 452g and 574g, respectively.

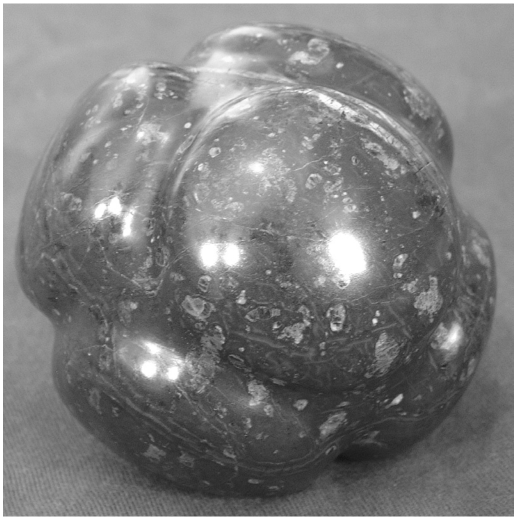
The fairly broad optimum in the maximum throwing range permits somewhat higher masses to be chosen for increased impact momentum, ie bone-breaking effect, with a negligible sacrifice in range. Thus this unconstrained pure physics analysis would seem to suggest that the Scottish carved stone balls are remarkably well-optimized as weapons intended to be thrown directly by hand. It should be noted that if higher density stone, lower speed, shallower launch angle and/or a lower drag coefficient had been selected, the optimum throwing range would favour somewhat lower mass than the curves shown (illus 9b). Because some of these factors are likely to be true of the Scottish carved stone balls and how they could have been thrown, the concept is reinforced that the actual masses chosen are more consistent with a function demanding both long throwing range and a heavy impact on the target rather than maximum throwing range alone. In principle, balls made of stone of differing density would have different optimum diameters and masses, higher density requiring smaller dimensions (or more material carved away). A statistical test of this hypothesis remains as possible future work, as it would require detailed measurements of the density (made difficult by the irregular shapes and varying porosity of the balls) and best-guess extrapolations to account for the volume of material lost to damage, quite significant in many examples.

As was found historically for golf balls, polished smooth spherical balls would have a high drag coefficient, $C_d = 0.5$, for the speed range achievable when thrown by hand. As noted above, the critical speed for drag reduction of such a polished sphere would be at least 60m/s (about 130mph) for a diameter of 70mm, much too fast for unaided throwing of something so heavy. It is primarily the unpolished rough surface of the Scottish balls that will keep their drag coefficient down for readily achievable speeds, while the density of the chosen material determines the best possible size and mass for throwing. The carved knobbles would only appear to confer any significant advantage to the aerodynamics when the surface roughness would otherwise be low, ie less than about 2% of the diameter. The striking resemblance of the deeply carved balls (illus 2) to the mace heads of later millennia highlights the idea that the knobbles are there to accentuate (focus) the impact on the target. However, it should be noted that a significant fraction of the balls found so far are simply rough, or much more lightly carved than this.

SPECULATION ON POSSIBLE USES OF THE BALLS

It would seem that these carved stone balls are well-optimized for, and thus were very likely intended for, direct throwing by hand. They are a delight to handle, but the roughness deliberately left on all bar a very few of the 200 examples handled in the pursuit of this work slightly detracts from this tactile experience. Perhaps the surface in early examples was originally left rough (and/or given grooves) to improve the grip of the ball when used in wet conditions, and it was found that rough balls could be thrown further than smooth ones. In any case, essentially they are all either rough due to a deliberately unpolished surface texture, or rough by virtue of fine-scale carvings such as cross-hatching. If they were intended solely for some application where a sophisticated tactile experience was paramount, surely far more would be highly polished like jadeite axe-heads, perhaps more resembling another one made in the course of preparation of this paper by the author (illus 10). This, made of red serpentine from a very much larger block, took about 25 hours to form with modern hand tools, and about 15 hours to sand and polish by hand. It will be evident that in surface finish it does not resemble the balls shown in illus 2 at all, only in the basic shape.

A few of the Scottish stone balls are very intricately carved, perhaps as though the owners wanted to be able to identify them amongst others. This may suggest that the application was for some form of competition. The great majority are not so decorated, however, and an obvious suggestion is that they were a kind of



ILLUS 10 A modern 85mm diameter, six-knobble ball made from red serpentine

throwing weapon, potentially quite effective on birds and small animals. This could be consistent with the observation that they are on the heavy side of the optimum diameter regarding range: with little sacrifice in range, a more effective weapon is achieved. In addition, as the surface roughness or knobble height only needs to be 1–2% of the diameter to be sufficient to 'trip' the boundary layer for all speeds of interest, the deeply carved knobbles are not necessary for this and are likely to have had some other function, most obviously that of focusing the impact of the ball on the target.

The balls would be likely to be damaged by impacting large rocks, so if they were weapons, a possible application could have been above cultivated or close-cropped fields. Many of the finds are not in any determinable archaeological context (Marshall 1976, 40–72) and as they are not easily distinguished from ordinary stones beyond about 10m from the observer (MacGregor 1999, 258–71), would quite readily have been accidentally lost if deployed in areas with many natural stones in evidence, for instance while hunting. Tidal strand-line bird hunting would seem to be ruled out, as the distribution of finds is strongly centred inland (Marshall 1976, 40–72; MacGregor 1999, 258–71), not near shorelines except in small numbers such as the finds at Skara Brae, Orkney. This would seem to make the most likely weapon-oriented function that of protecting crops or exploiting the attraction of small game birds and animals to cultivated fields. Another possibility is that they were used by shepherds protecting their flocks from predators, where a more accurate and effective weapon than the nearest natural stones to hand (and a weapon not very likely accidentally to kill the animals being protected) would be desirable. Also, arrows and spears cut hides, whereas blunt balls break bones internally but leave the skin intact, which may have been important if there had been ritual, functional or decorative applications for the skins of the targets.

Apart from the very few that were exquisitely carved, which may well have taken a week or so to make, the experience of the author and that of Sheridan (2005) is that the fabrication of a typical carved stone ball, to the standard of finish exhibited by the great majority of the Scottish carved stone balls, takes about 11–25 hours. This effort might well be considered to be practicable for something functional, while a great deal more effort (geometric accuracy, fine polishing, zones of contrasting finish perhaps) would surely be warranted for something intended to impress and to last.

Nevertheless, as remarked by Barclay (2003, 127–50), the distribution of finds of carved stone balls bears a striking resemblance to that of stone monuments, especially recumbent stone circles, heavily concentrated around modern-day Inverurie in Aberdeenshire and relatively lightly scattered elsewhere in Scotland. These are thought to be broadly contemporary and so while essentially optimum range when thrown by hand would seem to have been a feature of the original application of the balls, the function may have been associated with a localized cult rather than a Neolithic practice generic throughout the region now known as Scotland. A strong motivation for trial-and-error range optimization might have been stone-throwing as a test of manhood. If the last few per cent of range mattered, however, this should have led to characteristic masses nearer to 200g than 450g, so I am still inclined to the hypothesis that the balls were deliberately made a little heavier than would be optimum for range alone, ie as some kind of weapon.

Appendix 1 lists various possible applications of the Scottish carved stone balls, encountered in the references cited in this article, websites on the subject and discussions with personal contacts. They are listed in decreasing order of likelihood as perceived by the author, a physicist and engineer, rather than an archaeologist with training and experience in the social environment and belief systems prevailing in this area in Neolithic times.

CONCLUSIONS

Analysis, supported by simulation experiments with the relevant range of Reynolds number, reveals that the Scottish 'carved stone balls' of antiquity are strikingly close to the optimum size, weight and shape for throwing the longest possible distance by hand, unaided by any sling-shot or hurler. They are on the heavy side of the optimum for range alone, suggesting a weapon-like function of some kind. The significant roughness and/or the carved knobbles would promote low drag at modest speeds. If they were intended to serve a purpose confined to art or ritual, there would surely be a considerable size variation in the hundreds of specimens found, as seen today in the spectacular modern equivalents in Festival Square in Edinburgh. Surely also, if the function was beyond utilitarian, the surface finish would be generally developed to a high degree of symmetry and polish as in other Neolithic objects unequivocally identified as ritualistic or symbolic, such as prestige axe-heads.

Smaller balls can be thrown fast but have insufficient momentum to sustain the flight against air drag, larger ones are too heavy for a human to throw fast, so although air drag is then not very important, the throwing range is small. Somewhere in between lies an optimum size when thrown by a human, which turns out to be close to, but a little below, the size of the vast majority of the Scottish balls, 70mm diameter or 450g mass. I suggest that this 'optimization' is not a coincidence but the result of practical trial and error, much like that of golf balls a few thousand years later. Although possibly used for sport, these balls, slightly heavier and more knobbly than necessary for optimum range, would have been well suited for killing birds and small animals, or deterring predators such as wolves and eagles from attacking domestic flocks, in locations where the balls would neither be damaged nor lost upon landing, such as cultivated fields or relatively clear margins around them. Perhaps they were used by shepherds for this purpose, rather than weapons lethal to the herd animals, such as spears or bows and arrows.

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UNPUBLISHED SOURCES

Sheridan, A 2005 Private communication from researcher's report held in the NMS 'Old Customs House' archive store in Leith, Edinburgh.

APPENDIX 1

POSSIBLE APPLICATIONS OF THE CARVED STONE BALLS OF SCOTLAND

1. Free-thrown (weapon)
2. Single bola (Manrikigusari)
3. Free-thrown (max range)
4. Prestige goods
5. Story-telling totem
6. Decorative art
7. Gaming pieces
8. Water-spirit appeasement
9. Single bola (sling shot)
10. Mace heads
11. Currency
12. Fish stunner
13. Bolae
14. Divination tool
15. Standard weight
16. Almanac
17. Identity tokens
18. Funereal adjunct
19. Dice
20. Dance or procession instructions
21. Stamps
22. Astronomical guide
23. Standard length
24. Line spools
25. Platonic solids