The typological study of the structures of the Scottish brochs

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SUMMARY

A brief review of published work highlights the varying quality of information available on the structural features of brochs, and suggests that the evidence is insufficient to support the current far-reaching theories of broch origins and evolution.

The use of numerical data for classification is discussed, and a cluster analysis of broch dimensions is presented. The three groups suggested by this analysis are examined in relation to geographical location, and to duns and 'broch-like' structures in Argyll. While the method is concluded to be useful, an improvement in the quality of the data is necessary to improve the results and to aid interpretation.

INTRODUCTION AND REVIEW OF PUBLISHED WORK

The brochs of Scotland have attracted a considerable amount of attention over the years, from the antiquarians of the last century to archaeologists currently working on both rescue and research projects. Modern developments in the principles and techniques of excavation, though, have been applied to very few brochs. Several of the excavations of the past few decades have indeed been more careful research-orientated projects than the indiscriminate digging of the last century, when antiquarians seemed intent on clearing out the greatest number of brochs in the shortest possible time. The result, however, is that a considerable amount of material from brochs exists in museum collections, but much of it is from excavations carried out with slight regard for stratigraphy and is consequently of little use in answering the detailed questions set by modern hypotheses. Synthesis of the available material has been attempted nonetheless, but the evidence from a few individual excavations of recent times has inevitably been overemphasised. There is at present a resurgence of interest in brochs, particularly with the North of Scotland Archaeological Services on hand to carry out rescue excavations in the north, and it is becoming apparent that existing theories may require revision in order to accommodate the evidence which is now being discovered.

Brochs as a class have invariably been defined by reference to a few imposing sites which survive as towers. The characteristic architectural features of these sites are taken as representative of all the 514 or so brochs which have been recorded, but without excavation the detailed structure of most of these sites remains unknown. Inevitably, features are preserved in the tower sites which are lost in the more ruinous majority, and so it can only be assumed that they were common throughout the whole population of brochs.

Because the destructive forces of man and nature tend to start at the top and work down,

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the ground-level characteristics of brochs are most widely known. The massive, circular dry-stone wall, from about 2·5 to 6 m thick and with only a single entrance, may occupy up to 65 % of the total diameter which can be between about 14 and 23 m. Cells and galleries may be incorporated in the thickness of the wall in various combinations at ground level, and the interior courtyard may contain signs of domestic occupation such as hearths, wells and tanks for storing shellfish. There is evidence to suggest the division of this area into 'rooms' in the primary occupation phase of one recently excavated example (Hedges & Bell 1980, 90), and other sites may be reinterpreted in this way.

Evidence for what happens above ground level comes only from the surviving tower sites, but the close similarity of ground-plans suggests that most, if not all, brochs can be included by analogy. The purpose of this distinctive ground-plan would appear to be to provide a broad, firm base for a high wall. This base may contain a gallery at ground level or it may be solid, but a high dry-stone wall is achieved by building a double wall, tied together by horizontal lines of slabs which divide the 'cavity' into a vertical series of galleries between the inner and outer walls. Columns of increasingly smaller lintelled gaps above a ground level entrance help to dissipate the weight of the masonry over the entrance lintel, a device which was probably a direct result of the need to pierce a high wall without the assistance of arches.

The artefactual material recovered from brochs is not considered in this paper, since the associations between artefacts and structures are inadequate at present. The problems caused by this can be seen when 'inadequate early excavations, reporting at second hand on sites, and the failure to identify these objects for what they were' are advanced as reasons for the occurrence of saddle querns on excavated sites where they have not been recorded (Caulfield 1980, 136)—their presence can be assumed because the standards of recording were so low. Either artefacts or structures can be studied in isolation (MacKie 1965), but this is a rather unsatisfactory approach which is forced by circumstances. The situation will only improve after more excavation has been carried out using the full range of modern techniques and it will be interesting to see how the preliminary results of the structural classification presented in this paper will be affected as the data-base improves.

The poor quality of the artefactual information has not prevented attempts at structural analysis, however, and the first detailed account was published by Angus Graham in 1947. Table 1 shows the numbers of sites and of various internal features available for examination at that time; subsequent excavation has only improved on these details in a small number of cases.

TABLE 1
Summary of structural samples from Graham 1947

	Definite brochs	Possible total	Guard cells	Internal diameter	Basal galleries
Shetland	51	95	7	14	4
Orkney	42	105	13	16	4
North Mainland	169	227	41	67	2
West Coast and Inner Isles	28	49	15	20	9
Outer Isles	8	28	3	6	6
Central Mainland	6	8	5	4	0
Totals	304	512	84	127	25

Upper galleries—29
Stair bases—45
External defences—102
Comparable structures—38

The internal diameters were summarized for approximately circular sites only, the total number recorded being 132 (Graham 1947, 78). A number of conclusions were drawn from a statistical analysis of the internal diameters (*ibid*, 78–80):

(a) The measurements from each region are similarly grouped about their means.
(b) There are significant differences between the mean measurements from each of the different regions.
(c) Shetland, Orkney and the N mainland form one larger group, and the rest of the regions another; the means of these two groups differ significantly.
(d) Orkney may occupy an intermediate position between the groups with large and small mean internal diameters.

Graham concluded that rather than a number of different groups, 'two different races or strains in the species broch' could be determined, one mainly northern and one mainly western (*ibid*, 79).

Sir Lindsay Scott produced a three-fold classification of brochs by height, based on surviving remains and the assumption that scarcement ledges were designed to support the roof of the broch (Scott 1947, 10–15). He judged the initial height of the structure from the height of the walls which were still standing, and the amount of tumbled stone which lay around the site. One does not need to visit many brochs, however, to appreciate the fact that they have formed a readily accessible source of stone for subsequent builders, and even in remote areas have supplied stone for sheep fanks or shielings at least. Frequently the secondary structures have been built in or near the broch ruins, but occasionally the destruction has been more complete. Dun Garsin on Skye, for example, provided stone 'to build the revetment on the side of the road to Sligachan many years ago' (RCAMS 1928, no 482). The discussion following Scott's presentation of this paper to the Prehistoric Society shows some awareness of this difficulty (Scott 1947, 35–6), and Mr Angus Graham argued convincingly against a number of points, including the use of scarcements as roof supports. Although height may have been the most important consideration for the broch designers there is little to be gained from assessing this variable accurately for classification now.

In 1965 Dr Euan MacKie published a chart of broch dimensions in his important work of synthesis on the broch and wheelhouse building cultures, as well as tables showing the occurrence and dimensions of features not itemized by Graham. These and the data for the broch chart are summarized in tables 2 and 3, some of the data being taken from MacKie's unpublished PhD thesis (1973).

TABLE 2
Summary of features after MacKie 1965 and 1973

	Definite brochs	Possible total	Guard cells	Basal galleries	Stairs
Shetland	15	96	9	1	3
Orkney	14	104	13	4	7
Caithness	21	149	23	1	15
Sutherland	17	76	20	0	11
West Coast and Inner Isles	24	46	('West')	8	('West')
Outer Isles	9	28	(21)	2	(15)
South West Scotland	0	5		0	
Central Mainland	4	9	3	0	2
Totals	104	513	89	16	53

Door checks—62
Scarcements—47

TABLE 3
Summary of data used in MacKie's broch chart

	Sample size	Internal diameter		Wall thickness		Wall proportion	
		Mean	Stnd dev	Mean	Stnd dev	Mean	Stnd dev
Shetland	17	8·88	1·15	6·72	2·34	50·68	4·96
Orkney	14	10·03	1·27	4·00	0·51	44·61	4·07
Skye & Glenelg	18	10·48	1·00	3·58	0·32	40·90	4·46
Outer Hebrides	8	9·55	1·50	3·11	0·23	39·78	4·18
NW Coast	2	10·30	0·70	3·60	0·00	41·40	1·40
Tiree & Mull	5	9·64	1·13	3·52	0·86	42·10	7·55
Caithness	27	9·14	1·38	4·18	0·48	47·85	4·95
Sutherland (E)	13	9·18	1·50	4·48	0·78	49·38	5·08
Sutherland (N)	11	8·75	0·83	4·18	0·77	48·53	5·17
Central Mainland	5	11·74	2·76	5·34	0·71	48·20	6·62

The chart appeared in revised form in 1971 (MacKie 1971, fig 1), where it showed on a scatter diagram the values of the following variables for a sample of 120 brochs:

(1) The variation of wall proportion against internal diameter.
(2) The geographical location of each site, in one of nine regional groups.
(3) The structural classification of each site as solid-based, transitional, ground-galleried or unknown.
(4) The absolute wall thickness of each site, shown on diagonals between the two main axes.

The variable 'wall proportion' is itself a ratio of total wall thickness against total diameter, giving a measure of the 'massiveness' of each broch. Dotted lines surround the geographical groupings of sites in Shetland, Orkney, and the W coast and isles, but there is no attempt to differentiate other regions among the generally overlapping distributions in the centre of the chart. Apart from a few omissions and mislabellings, a great deal of information is contained in the scatter diagram, making visual analysis difficult. On the basis of a subjective interpretation, MacKie draws the following conclusions (*ibid*, 41):

(1) Broch structure varies with location within the broch province.
(2) Brochs on the W coast tend to have:
    (a) thinner walls
    (b) larger internal diameters
    (c) ground galleries
(3) Brochs in the north tend to have:
    (a) thicker walls
    (b) smaller internal diameters
    (c) solid bases.

In order to construct hypotheses from these conclusions, it has to be assumed that the broch builders would improve their design with time, and that this improvement would lead in the direction of greater stability of the high broch wall. The resulting hypothesis describes the development of brochs from relatively unstable ground-galleried forms to more stable solid-based types (MacKie 1965, 128). The evidence to support this is slight, consisting of secondary walling in a few ground-galleried brochs which was apparently required to buttress the double wall (Graham 1947, 61). The Royal Commission claims, on the other hand, that ground-galleries are designed to increase stability on uneven ground (RCAMS 1980, 24), and there is also likely to be a connec-

tion between stability and the skill of the builders and the quality of the building material. Several brochs and broch-like structures in Argyll show signs of secondary buttressing (*ibid,* nos 166, 171 & 185 for example), and this may be designed to serve the same purpose as the internal revetment which strengthens the walls of a number of other sites. There are no signs of a gallery in the walls of An Caisteal on Mull (no 171), indicating that solid-based walls may also on occasion require buttressing. The sites shored up in this way are frequently located on restricted areas at the tops of precipitous slopes, which may be one reason for the need to reinforce the original masonry at some stage after construction. Perhaps there is a need for some empirical research into the stability of dry-stone structures in order to solve this problem, since experts in building stone walls on this scale must be rare today.

MacKie also suggests an increase in the sophistication of various design features, including the distance at which doors were placed along the entrance passage. The improvement comes in response to the threat of attackers using battering rams, and not only increases the difficulty of wielding the ram but also allows the defenders to counter-attack from the first-floor gallery above the entrance passage (MacKie 1965, 109). Not all brochs are provided with accessible space above the entrance, however, and in such cases it would be a positive disadvantage to create 'dead ground' right against the only weak point in the defences. The attackers would be able to concentrate on breaking or burning down the door, whilst completely sheltered from any defensive action.

The basic similarity of broch ground plans has suggested that groups of professional builders or designers carried the idea through the province, thus providing an element of continuity as a medium for design improvement. Again the evidence is tenuous, particularly when seen against the long tradition of drystone building which existed throughout the Atlantic Province (eg Ritchie & Ritchie 1981, figs 22 & 145).

MacKie's general hypothesis, therefore, is that the western brochs are the earliest forms, and that the more stable, solid-based brochs of the north developed from them over a period of time. The broch of Mousa, on a small island off the Mainland of Shetland, is the best-preserved monument of this kind, and is taken to represent the culmination in the north of the development of broch-building skills. These theories are based solely on a consideration of the structure of 120 brochs, out of a total of over 500 known sites, and MacKie's theory of west to north development is supported only by additional structural evidence and by the presence of possible prototype structures in the west, dated somewhere in the last three centuries bc (MacKie 1971, 42–3).

Further structural data would be difficult to collect without excavation, since the remaining sites are often little more than grassy mounds with hardly any internal features visible at all. In some cases when the walls are visible, it is only possible to measure them at some distance above ground level because of the rubble and earth which partially covers the site. The technique of 'battering' a stone wall causes it to become thinner with increasing height, a characteristic feature of broch walls, and this, in addition to constriction at a scarcement ledge and any slumping or collapse, makes the ground-level dimensions of a partly buried site almost impossible to determine. At this stage, however, it is more important to emphasise the chronic lack of associated cultural and chronological information; until knowledge in these areas is improved, hypotheses of structural origins, development, and even function will not be able to rise much above the level of speculation.

The numerical data which describe the structure of the 120 sites in MacKie's sample provide a starting-point for the construction of a numerical typology of brochs, and for the extension of this to investigate seriation within the resulting classification. However, only two variables, diameter and wall thickness, are available to express the dimensions of each site. Combinations of

these can be produced mathematically, such as external diameter, total wall thickness or wall proportion, but none of these adds any new information to that which is implicit in the two basic measurements. There is little chance of being able to ascertain accurately the variable which may originally have been most significant of all—height. The small sample sizes and poor recording of the positions of mural cells prevent these from being added to the list of diagnostic features (Graham 1947, 57), and the occurrence of lengths of basal gallery can be used to distinguish little more than the structural type of a relatively small number of sites at present. Internal diameter, wall thickness and structural type, therefore, provide only three descriptive variables, a barely sufficient number for a rigorous classification. An improvement in the quantity and quality of artefactual evidence would not only provide more data for this type of analysis, but would also give an important independent check on the results obtained.

The data for the sample of 120 brochs is summarized in table 4.

TABLE 4
Summary of variables for sample of 120 brochs

		Min	Max	Range	Mean	Stnd dev
1	Internal diameter	6·10	16·80	10·70	9·51	1·51
2	Wall thickness	2·40	6·30	3·90	4·12	0·73
3	Wall proportion	33·70	61·60	27·90	46·36	6·17
4	Total diameter	13·00	27·40	14·40	17·74	2·05

The scatter on MacKie's 'broch chart' (1971, fig 1) shows the approximately linear relationship between internal diameter and wall proportion. However, when the two basic variables are plotted together (fig 1) the lack of correlation between them is shown up, suggesting that the relationship shown in the 'broch chart' might not be as significant as it appears at first sight. Since 'wall proportion' is a ratio of wall thickness and internal diameter, some of the correlation of $-0·669$ is caused by the internal diameter being represented on both axes. The correlation of $0·768$ between wall thickness and wall proportion is high for similar reasons. The implication of MacKie's scatter diagram is that brochs with larger internal diameters tend to have thinner walls, but in fact the correlation of $-0·067$ between the two basic variables shows only a weak trend in this direction. The approximate relationships between these variables are shown by the following regression equations:

$$d = 10·08 - 0·07(W) \tag{1}$$
$$D = 10·08 + 0·93(W) \tag{2}$$

where d = internal diameter, D = total diameter, and W = total wall thickness. For every one metre increase in internal diameter, the (total) wall thickness tends to decrease by only 0·07 m, but for every one metre increase in total diameter the wall thickness tends to increase by 0·93 m.

This confirms that larger brochs in the sample will tend to have larger courts, and the more gradual decrease in wall thickness results in the drop in overall wall proportion—a factor which may quite simply reflect the limitations of the building material and the time and effort available. Presumably the broch builders sought optimum dimensions for their available resources, so that the effort of construction was not too great, and the space enclosed was not too small for the function intended for it. A considerable range in external diameter can be seen from table 4, the smallest broch in the sample being less than half the diameter of the largest. To maintain wall proportion from a small site to a large one would necessitate a considerable increase in the
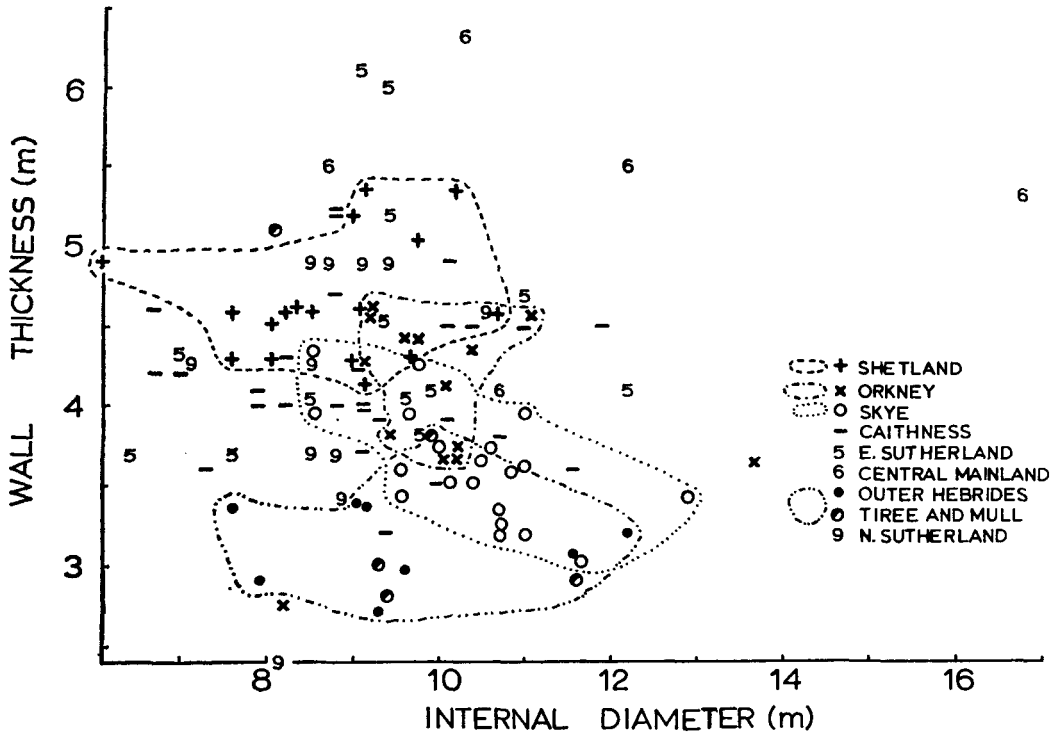
FIG 1 Scatter diagram of the dimension of 120 brochs, with regional groupings

amount of stone needed, and the time and effort involved. A very rough calculation shows that a mere 1 m increase in an internal diameter of 8 m would require an increase in the region of 100 m² in the area covered by the wall, in order to keep the wall proportion approximately constant. This may have been offset to some extent by a larger work force (which may reflect a larger group of people associated with the site and therefore, possibly, the need for a larger site in the first place). At some point, though, the expenditure would become uneconomical, and the thickness of the wall would have to be decreased. With a range of 10·7 m in the internal diameters of brochs in the sample, these practical considerations must have had some effect. It should be noted that at this stage the direction of development is not intended to be taken spccfically: the same argument is true for a decrease in internal size allowing an increase in wall thickness. Unfortunately, this provides no more than circumstantial evidence concerning the height of broch walls. With a larger diameter, the stability of the wall could presumably have been improved by increasing the thickness of the base.

The first version of MacKie's scatter diagram (1965, fig 3) showed wall proportion against external diameter, but the correlation between these two variables is low. The relationships between total diameter and the basic variables internal diameter and wall thickness are clearer, with internal diameter increasing rather more rapidly than wall thickness. In addition to diameter and wall proportion, MacKie's broch chart also shows the regions in which the sites are found. MacKie comments that 'the total external diameters do not separate out convincingly', but 'the relative massiveness of the wall bases does' (ibid, 105). The revised broch chart (MacKie 1971, fig 1), which shows internal diameter instead of external on the x-axis, does not in fact alter the

relative positions of the sites very much. The most noticeable regional groups consist of the dozen or so Skye brochs with low wall proportions and the Shetland sites which are restricted to the upper half of the diagram. Other regions tend to be more widely spread: Orkney, the Outer Hebrides, Tiree and Mull mainly in the lower end of the range, Sutherland mainly in the upper reaches, and Caithness sites throughout both.

The plot of internal diameter against wall thickness shows a similar pattern (fig 1): Outer Hebridean sites occupy the thin-walled area of the chart, although most of the range of common internal diameters is covered. Skye brochs overlap with the thinner-walled sites in the Northern Isles, and the intermediate position of the Orcadian sites is clear. If it were not for three or four outlying cases the Orkney brochs would cover the full range of Shetland wall thicknesses and Skye internal diameters; the broch of Oxtro has a greater internal diameter than any W coast site, and is second only to the Central Mainland site of Edinshall in this sample. Redland, another outlying Orcadian broch, has the third thinnest walls of the sample. Most of the Caithness brochs occupy a central position on the wall thickness scale, but cover a wide range of internal diameters; Sutherland sites are widely spread throughout.

This re-arrangement of the data into a more readily understandable form involves no extra information, and serves to clarify rather than extend the conclusions which have already been drawn. The small sample sizes from the various regions do not allow much detail in these conclusions, and this can only be improved by future excavation. If structural similarities can be reinforced by dating and artefactual evidence, it may ultimately prove possible to build up a convincing picture of how the broch design developed throughout the Atlantic Province. At the moment, however, even a simple objective view of the data warns against too much speculation based on too little evidence.

THE USE OF NUMERICAL CLUSTERING TECHNIQUES

The techniques of numerical taxonomy have come to archaeology from subjects such as psychology and zoology, and have been found to be of use along the way in subjects as diverse as astronomy and bacteriology. Consequently, a considerable literature already exists on a variety of levels (eg Clifford & Stephenson 1975; Sokal & Sneath 1963; Sneath & Sokal 1973) and as in other areas where techniques are being borrowed from other disciplines (Martlew 1981), archaeologists would be well advised to familiarise themselves with the cautionary tales which have already appeared in print.

Despite the usual advantages of numerical techniques over subjective interpretation by eye, a number of problems exist in cluster analysis which if anything reintroduce subjectivity, but at a different stage. Most clustering techniques seek to identify a particular structure in the data; if there is no structure present an incautious approach may force one onto the data, and if the structure is different to that which is being sought, it may be missed altogether (Everitt 1974, 77–97). Of the two basic approaches to classification, most techniques are suitable for breaking a continuous distribution down into discrete groups (dissection). Similar problems can arise, though, to those which exist in the other basic approach, when a classification is being sought by joining individuals to existing or suspected groups within the data. It is possible for different techniques and different treatment of the data to produce different results, so that false classifications may arise if a careful strategy is not followed.

In addition to these problems there is controversy over the use of formal statistical tests to show the significance of particular groupings of the data. While the application of a single clustering procedure is probably insufficient to produce a convincing general classification, there

is a suggestion that assessment of the results may inevitably contain an element of subjectivity, in the absence of clearly defined statistical measures of significance (*ibid*, 74, 96). The availability of computer software packages which perform cluster analyses rapidly and easily may only help to exacerbate these problems if due care is not taken. The computer will do all the calculations regardless of whether an appropriate analytical strategy is being followed, and will pour out pages of results which the archaeologist still has to interpret in terms of his or her own subject (Hodson 1980). Awareness of the problems, however, will help to keep the results in perspective, and will enable a useful technique to make an important contribution to the solution of a wide variety of archaeological problems.

Over the last decade or so several archaeologists have used techniques of multivariate analysis in order to produce typological groupings, mainly of artefacts, based on numerical data. Various approaches have been tested (Hodson, Sneath & Doran 1966; Hodson 1969; 1970), and cluster analysis, amongst others, has been applied in a number of different fields (eg Green 1975; Allsworth-Jones 1975). The data on the sample of 120 brochs can be examined objectively in this way, and the results compared with the conclusions drawn from a visual examination of the scatter diagrams. The analysis was carried out using the Clustan 1B package, on the North West Regional Computing Centre's CDC 7600 computer at Manchester (Wishart 1979).

The data were initially standardized to prevent undue bias from extreme cases, and the first procedure chosen was Ward's method of hierarchical fusion (Ward 1963), using a similarity coefficient of squared Euclidean distance between individuals and clusters (also known as the k-means approach (Hodson 1970, 311)). Groups of cases with a minimum of variation are determined, based on the distances of each individual from the centroids of the clusters (Wishart 1979, 9 & 31). Individuals and then groups of individuals which are most similar to each other are gradually joined together, a process which can be represented by a dendrogram included in the print-out (Clifford & Stephenson 1975, 138–42). The dendrograms reproduced here (figs 2, 3, 4, 7, 8) have been re-drawn from the computer output; because of the reduction necessary for publication it has proved impracticable to include the 120, 156, or 168 case numbers clearly on the diagrams. The general shape of the dendrograms is perhaps more important at this stage, but copies of the originals may be obtained from the author on request.

Fig 2 shows the dendrogram for the run on the broch structural data, and the close similarity between most of the sites can be seen by the way in which they are combined at a low coefficient of dissimilarity, except for site 118. This is the Central Mainland site of Edinshall, the largest site in the sample and one of only six out of 132 measured examples where the major and minor axes differ by more than 6% from their mean (Graham 1947, 77). In the case of any such discrepancy, mean dimensions are used throughout this analysis.

Because the spherical clusters found by Ward's method may have been created by the technique rather than detected by it, several alternative procedures in the Clustan package were also applied to the data (Wishart 1979, 30–1). The single linkage (or nearest neighbour) technique did not produce satisfactory results since the chaining effect gave a large number of small clusters, most of which were combined at a low coefficient of dissimilarity (Hodson 1970, fig 2). Several atypical sites including Edinshall, Tappoch, Torwoodlee and Mousa were joined on individually at slightly higher coefficients, and provided the most recognizable structure in the dendrogram. There is some controversy over the use of single link cluster analysis (Everitt 1974, 67–9; Doran & Hodson 1975, 176), and it does not appear to be able to detect any significant structure in this particular data set.

Complete linkage (or farthest neighbour) cluster analysis suggests a two- or three-cluster grouping (fig 3), neither as pronounced as the three-cluster grouping produced by Ward's method.
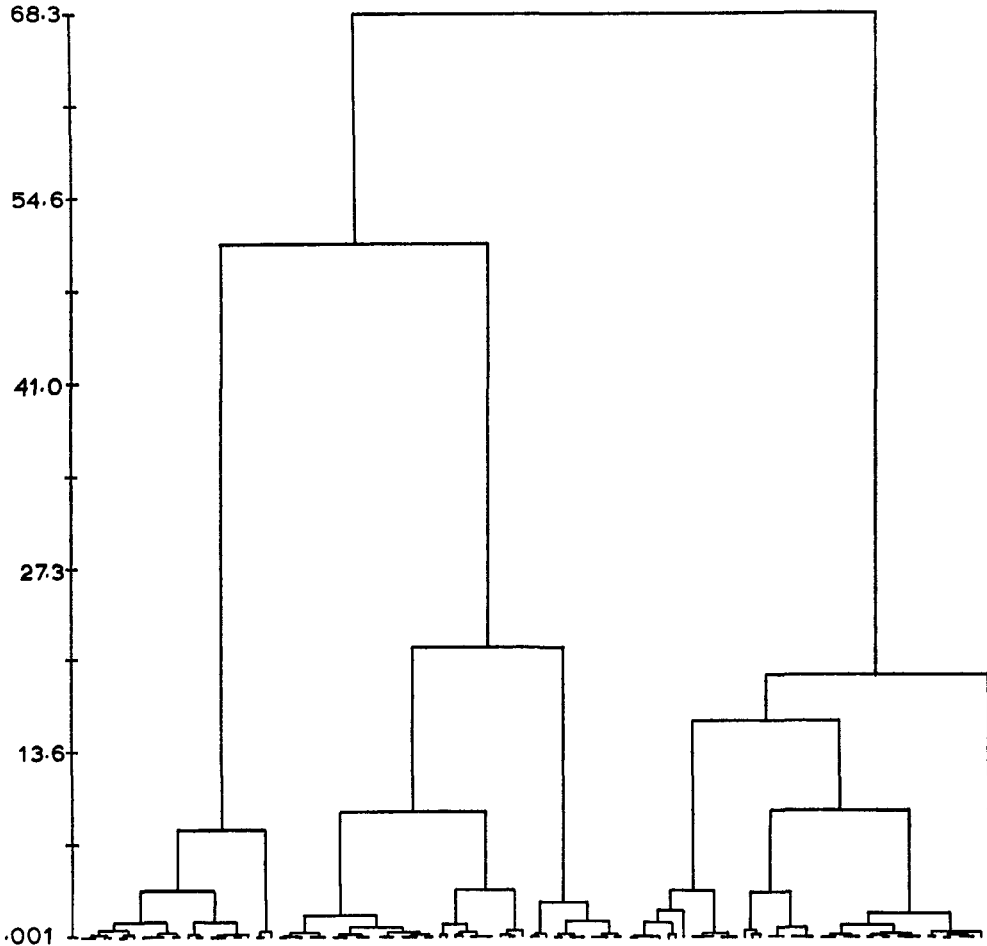
FIG 2 Dendrogram showing hierarchical fusion of 120 brochs based on internal diameter and wall thickness. Edinshall (no 118) is on the extreme right; vertical scale shows increasing dissimilarity

A small group of 13 sites consisting of brochs with larger than average internal diameters and wall thicknesses is combined with two major groups, but Edinshall remains on its own long after all the other sites have been reduced to one group. This technique is also open to problems (Wishart 1970, 30), and has not been used as often as other methods.

Single and complete linkage clustering techniques compute similarities on the basis of nearest and farthest neighbours respectively, selecting one individual only to represent each cluster. Average or group average analysis works on the average similarity coefficient of all the pairs of individuals from two clusters. Although, unlike the previous two methods, it can deal with structure within the data, it has not been found to be suitable for producing definitive results (Doran & Hodson 1975, 177); with the present data set it suggests the formation of one large group with the successive addition of two or three smaller ones (fig 4).

None of these techniques suggest a grouping of the data with the same clarity as Ward's method, although complete linkage suggests a similar structure. Because of the various mathe-
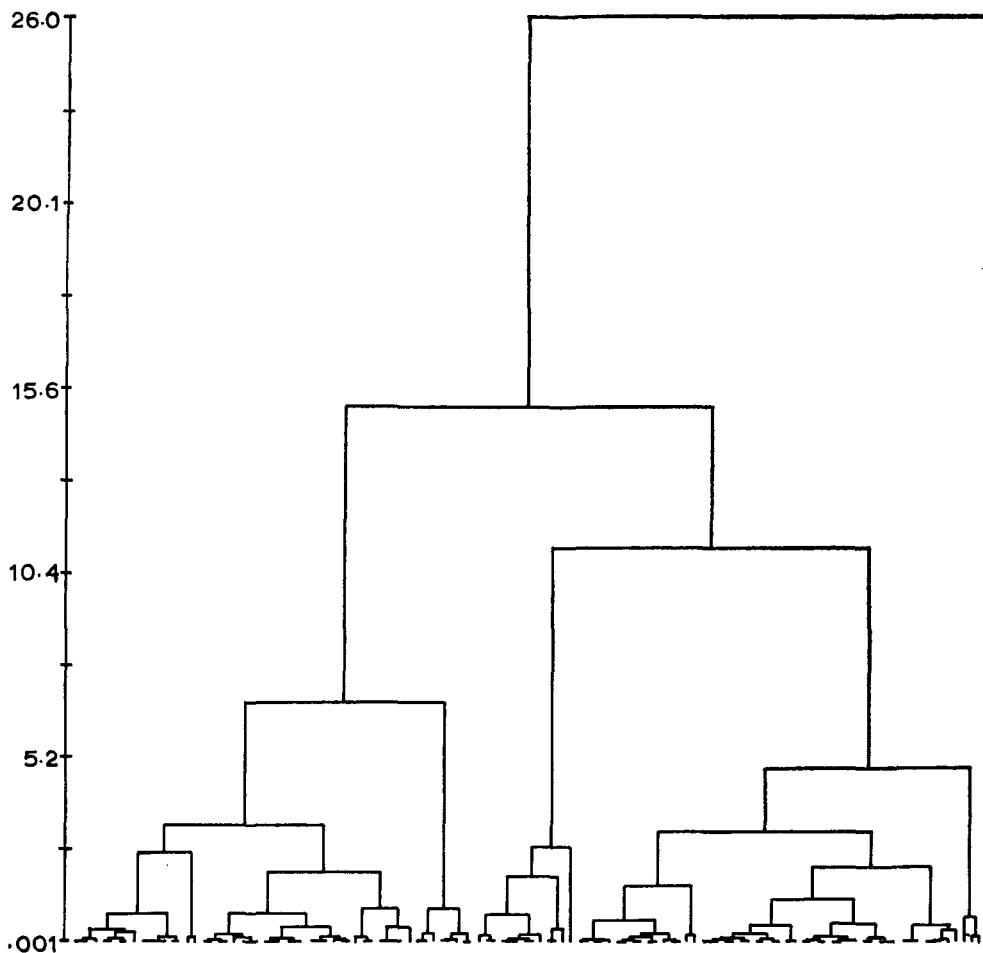
FIG 3 Dendrogram produced by complete linkage cluster analysis of the dimensions of 120 brochs

matical difficulties (Hodson 1970) Ward's method, the k-means approach, is to be preferred, and this analysis seems to support its superiority. The other techniques, though, may be allowed to sound a note of caution despite their theoretical problems, emphasizing the need for improvements in the collection of the data and the choice of variables available for analysis.

INTERPRETATION OF THE RESULTS

The dendrogram for Ward's method of hierarchical fusion shows three main groups which appear at coefficients of 7·9, 19·4 and 21·4, and which are not combined until coefficients of 51·2 and 68·3 are reached. The analysis was run on the variables internal diameter and wall thickness only, since any combination of these would not add new information. The three clusters are shown on the scatter diagram (fig 5), but comparison with fig 1 shows that it is not easy to pick them out by eye. Various procedures can be followed for checking the analysis internally. Iterative relocation of individuals from eight random clusters gave the same results as from the eight
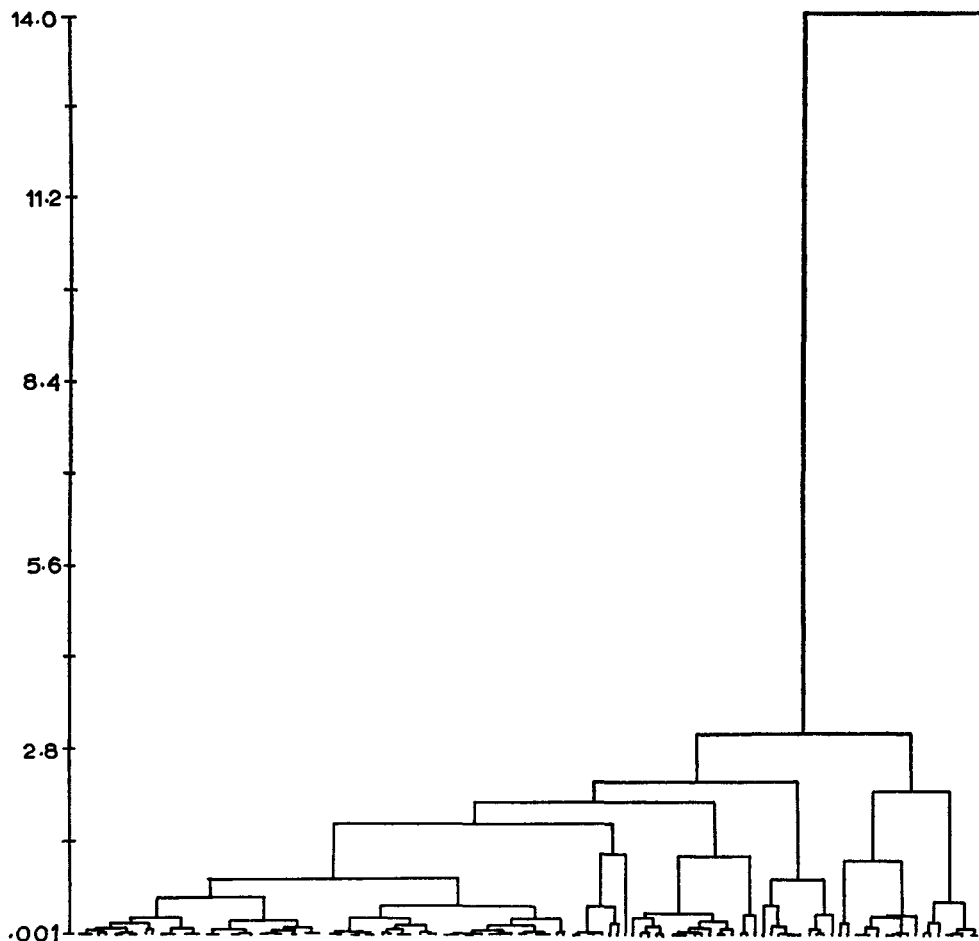
FIG 4   Dendrogram produced by average linkage cluster analysis of the dimensions of 120 brochs

cluster stage of Ward's method, which confirms that the measure of similarity is valid (Wishart 1979, 9). The three cluster grouping produced by this method is, however, slightly different to that produced by Ward's method, and a really robust classification would obviously require more information.

   The cases listed in the output as being least similar to their parent cluster, with a dissimilarity coefficient greater than an arbitrary threshold of 1·0, are perhaps the ones which deserve most attention: more information may shift the centroid of the cluster in which they have tentatively been placed, but there may be other reasons for their eccentricity which may repay closer study or even excavation. It is interesting to note that these atypical cases include such well known sites as Mousa, which is often described as generally representative of all brochs, and Dun Carloway, another broch which survives to a considerable height, on the Isle of Lewis.

   In fact all eight Outer Hebridean brochs lie on or near the fringes of their cluster, and more data from this region may enable a new group of thin-walled brochs to be formed. Four of the five Central Mainland sites are located at the opposite extreme, with Edinshall, Torwoodlee and
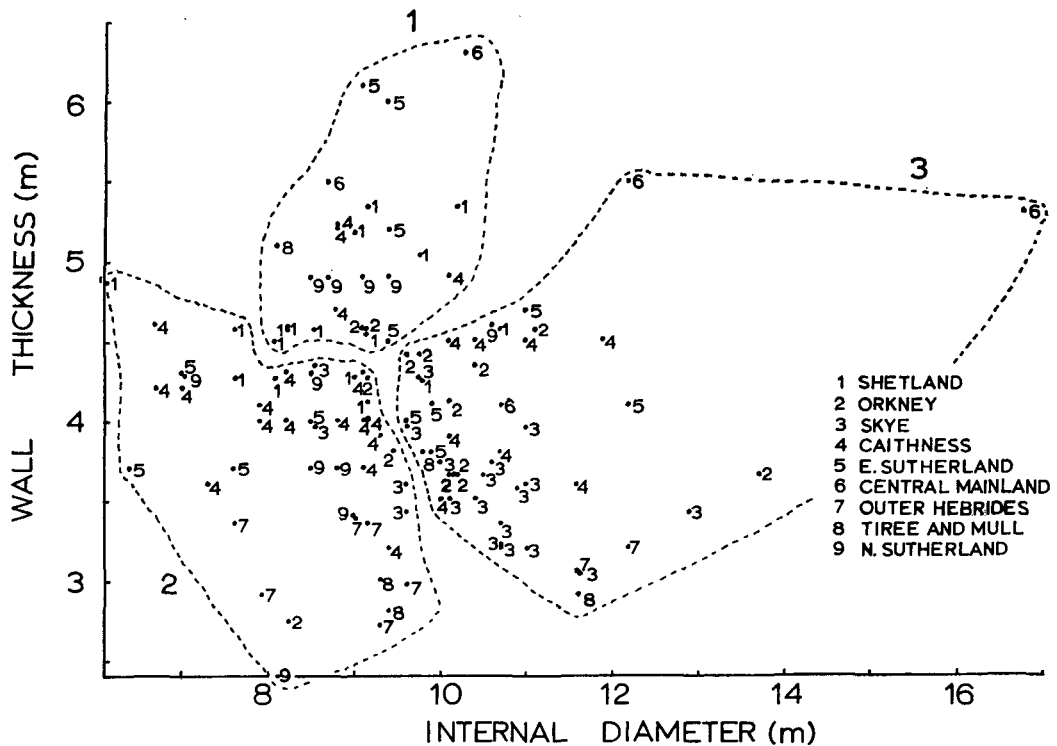
FIG 5 Scatter diagram of 120 broch dimensions showing the three-cluster grouping produced by Ward's method of hierarchical fusion

Tappoch lying well away from the main concentration. At least three of the 24 eccentric sites are well-preserved brochs, surviving to the first upper gallery and above (Graham 1947, 62). However, no confidence can be placed at this stage in the strategy by which the sample was collected, and it may well be that future work will show it to be unrepresentative. This being the case, it is unwise to place too much emphasis at the moment on seemingly atypical cases, but this aspect will undoubtedly prove to be worth further investigation.

Despite the cautious approach adopted towards the interpretation of these results, the diagnostic statistics printed by the computer for each cluster suggest that there is little variation within clusters (table 5a), and show how each differs from the population mean (table 5b).

The means and standard deviations of the variables in each cluster are shown in table 6.

The amount of variation covered by each cluster can easily be seen in fig 5, where it becomes obvious that the high variation in cluster three is largely a result of the presence of the two

TABLE 5
F-ratios and T-values for the three cluster grouping

	(a) F	Var 1	Var 2	(b) T	Var 1	Var 2
	1	0·155	0·446	1	−0·307	1·261
Cluster	2	0·385	0·580	2	−0·750	−0·437
	3	0·673	0·588	3	0·885	−0·265

Expected values under random sampling: F=1·0, T=0·0

TABLE 6
Cluster means and standard deviations

		Means	Var 1	Var 2	Stnd dev	Var 1	Var 2
		1	9·05	5·04	1	0·60	0·49
Cluster		2	8·38	3·80	2	0·94	0·56
		3	10·85	3·92	3	1·24	0·56

eccentric Central Mainland sites, Torwoodlee and Edinshall. The table of T-values shows an interesting corroboration of hypotheses deduced from the scatter diagram and the regression analysis. Cluster one is characterized by smaller than average internal diameters, and thicker than average walls; in cluster three the variables show the reverse, while in cluster two both are smaller than average.

Perhaps the most interesting aspect of this numerical analysis, however, is that data on geographical location and structural type were included, so that each cluster could be examined on this evidence also (although these binary variables could not be included in the computation of similarities based on diameter and wall thickness). Table 7 shows the actual number of sites from each of the six main regions in each cluster, and fig 6a shows these as percentages of the total number of sites in the cluster (the right-hand total column in table 7). This picture is somewhat distorted, though, by the wide variations in sample size, from five in the Central Mainland to 33 in the West Coast region. In order to make some allowance for this, fig 6b shows the number of sites from each region in each cluster as a percentage of the total number from that region (the totals on the bottom row of table 7). Some interesting details can be picked out, such as the large percentage of Shetland sites in cluster one and Orcadian sites in cluster three, but again the lack of sampling control requires the results to be interpreted with caution.

TABLE 7
Numbers in each cluster from the six main regions

			Region						
		1	2	3	4	5	6	
	1	9	2	1	4	8	2	26
Cluster	2	6	3	12	15	10	0	46
	3	2	9	20	8	6	3	48
		17	14	33	27	24	5	120

It does seem possible, however, to recognize a major geographical component in each of the three groups: cluster one is dominated by Shetland and Sutherland brochs, the latter being equally divided between sites from the east and north of the old county area. Cluster two contains large numbers of sites from Caithness, with a secondary group from the W coast which in fact contains all except two of the Outer Hebridean brochs. The third largest group in this cluster is composed of Sutherland brochs with a slight tendency towards northern sites. Cluster three contains 15 of the 18 Skye brochs, and also the highest percentage of Orkney brochs.

Despite the problems of initial sampling and sample size, this classification of the 120 brochs seems to support the hypothesis that broch structure varies from one region to another. The tendency indicated by the cluster one T-values for small internal diameters and thick walls accords with the predominance of 'massive' sites in Shetland; in cluster two both variables tend to be less than the sample mean, defining a group of smaller, less 'massive' sites in the N mainland and the west, while the third cluster presents western and Orcadian sites with larger internal
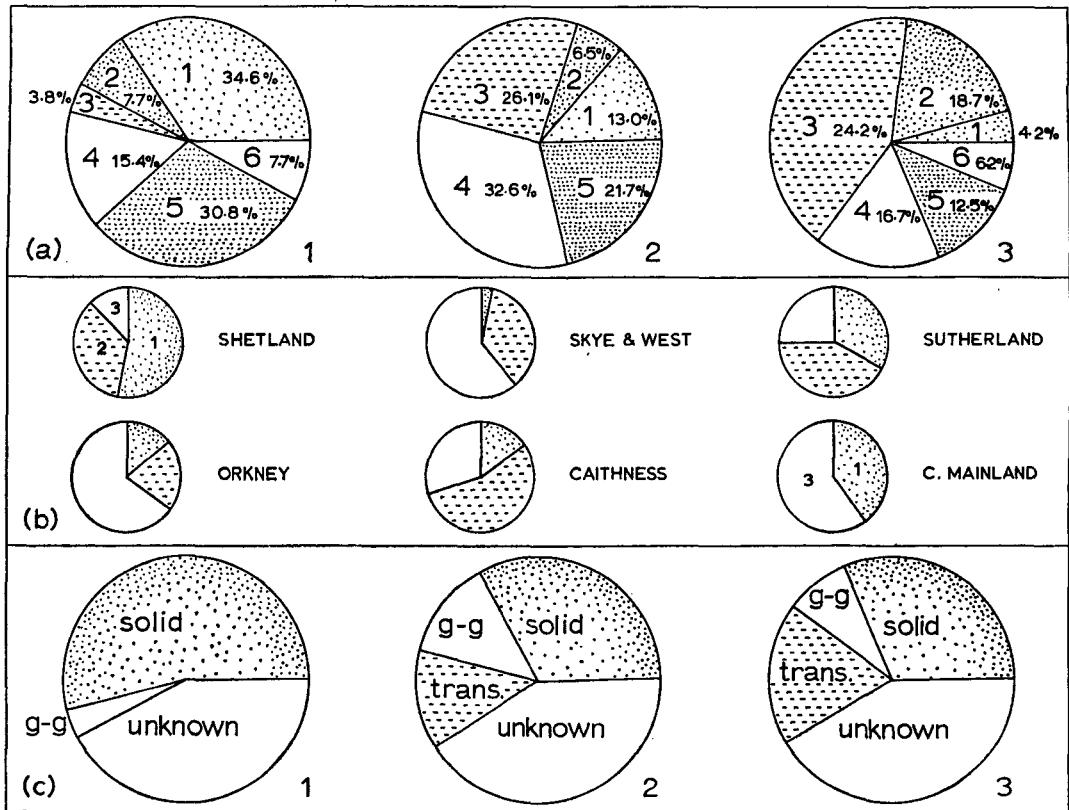
FIG 6 Numbers of sites from each region in the three clusters produced by Ward's method: (a) as a percentage of the cluster total. (b) as a percentage of each regional total. (c) the percentage of structural types in each cluster

diameters and thin walls. Modern geographical names can therefore be used to identify the three clusters, as follows:

cluster one: Shetland/Sutherland group
cluster two: Caithness/Outer Hebrides group
cluster three: Skye/Orkney group

The significance, and indeed the confirmation, of these results must await the completion of much more work in the field. The addition of 31 sites, albeit only approximately measured by the Royal Commission and not checked subsequently, shows how the inclusion of new information at this stage can affect the classification produced (fig 7). From the three-cluster stage a few of the larger sites (including Edinshall) are merged to form a dichotomous grouping: cluster one, the larger of the two, consists of sites with small internal diameters and thicker than average walls (T-values of -0.42 and 0.42 respectively), and cluster two contains the reverse (T-values of 0.75 and -0.74). There is an almost exclusive geographical division between the north and north-east in cluster one, and the west in cluster two (including sites in Argyll and Wigtownshire).

Although this emphasizes the need for unbiassed sampling, it also shows one of the advantages of this method in typological studies. Instead of tacking the new data on to existing groups
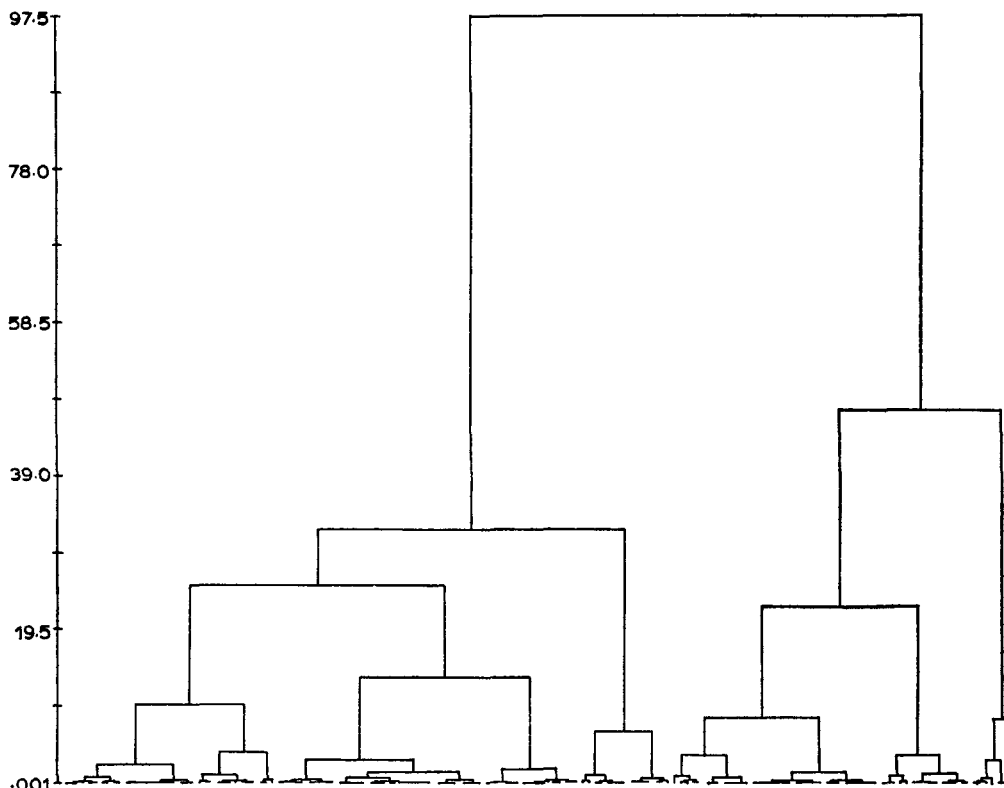
FIG 7   Dendrogram produced by Ward's method of hierarchical fusion of the dimensions of 156 brochs

the whole analysis is run again, producing taxa from a simultaneous consideration of all the data available. Only when rigorously defined groupings have been arrived at can techniques such as discriminant function analysis be used to place new individuals into the appropriate class using the same objective numerical data (Lachenbruch 1975).

Another important consideration is how the various 'comparable' or 'broch-like' structures will ultimately fit into the picture. Galleried duns such as Ardifaur (Christison 1905) and Dunburgidale (Hewison 1893) are separated from the main broch class by their large internal diameters (19·8 and 20·4 m respectively), but they both possess many of the 'characteristic' architectural features of true brochs, and their walls are no thinner than those of several brochs at the lower end of the range. Not all 'broch-like' structures, though, are so easily differentiated on the basis of dimensions alone, and where they overlap in size the criteria must be shifted to such features as entrances, galleries, and circularity of ground-plan, in the total absence of any associated cultural material.

The irregular lay-out of most galleried duns makes it difficult to reduce their dimensions to 'internal diameter' and 'wall thickness' for direct comparison with brochs on the scatter diagram, but it is apparent that even using coarse mean dimensions (RCAMS 1928) the galleried duns on Skye begin to fill in the gap between the thin walled brochs and the much larger sites of Ardifaur and Dunburgidale. The 'semi-brochs', which Beveridge identified as a separate class on Tiree (1903, 73–83), are in some cases no more than ground-galleried brochs, as the excavation of what Beveridge called (*ibid*, 76) 'the most distinct specimen in Tiree of the semi-broch type',

Dun Mor Vaul, has shown (MacKie 1974a). The other 'semi-brochs' on Tiree are more reasonably included in the general class of 'broch-like' structures, rather than being given a category of their own. Beveridge does recognize the important criteria, though, when he links his semi-brochs with the brochs on the basis of their 'symmetrical and massive circular construction' (*ibid*, 162). Few of the galleried duns show the same circularity of ground-plan as the brochs, but this is masked by considering only the two mean dimensions internal diameter and wall thickness. In no more than six out of 132 internal diameters examined by Graham did the major and minor axes of a broch differ by more than 6% from their mean (1947, 77), holding out the hope that it may eventually be possible to incorporate this measure of circularity into the definition of brochs. The most obvious case in the present analysis is Edinshall, which on internal diameter and wall thickness alone is less like a broch than a good many galleried duns are. In fact the 8% discrepancy from the mean of the axes, which only represents just over 1 m on the ground, could be used to group it with the circular brochs.

The work of the Royal Commission in Argyll (RCAMS 1971; 1975; 1980) has added a further dimension to the consideration of 'broch-like' structures. A number of duns have been recorded with mural galleries or cells, and at least 13 of these sites possess two or more architectural features which are characteristic of brochs (RCAMS 1980, 24–5). It is interesting to note that a number of these sites which are not classified as brochs because of their lack of circularity are built on very restricted locations, and presumably the oval or irregular ground-plans are attempting to make the most of the small amount of relatively level ground available. The circularity ratings of these sites lie between 7 and 15%; those of the galleried duns on Skye are between 10 and 26%, although in some cases it is impossible to be certain of the effect of erosion on these cliff-edge sites. Dun Ardtreck, the 'semi-broch' excavated by Dr MacKie, may conceivably have been constructed as a complete circuit, as has been suggested for Dun an Ruigh Ruadh (Calder & Steer 1949). At Dun Ardtreck the present 'open straight edge formed by the cliff' (MacKie 1967, 27) may be due solely to erosion of the western side of Ardtreck Point by the sea, although sites like Ruadh an Dunan are more likely to have been promontory defences than completely enclosed cliff-edge sites.

Adding 12 'broch-like' structures from Argyll (RCAMS 1980) to the extended broch sample restores the three-cluster grouping with the same relative dimensions as the initial run on the sample of 120 brochs, but with a slight shift in geographical emphasis (fig 8). Cluster one now contains almost all the Shetland sites and the majority of Caithness and Sutherland brochs. Orcadian sites are split between this group and the second largest cluster, which is dominated by Skye brochs and contains just over half of the 'broch-like' structures, in addition to some of the south-western brochs. The cluster means indicate small courts and thick walls in cluster one, and the converse in cluster two. The third group consists of brochs with smaller than average dimensions, and contains all except two of the Outer Hebridean sites; a few are present from most other regions, along with three 'broch-like' sites from Mull and one from Tiree. A certain amount of stability is apparent in these repeated classifications, with a W/NE dichotomy and a group of smaller sites in the Outer Hebrides collecting similar-sized brochs from elsewhere. Despite the irregularity of ground-plans, the analysis indicates that on these dimensions alone the 'broch-like' structures of Argyll are more like brochs than some sites accepted as such elsewhere.

In the absence of dating evidence in particular, it is difficult to do more than speculate about the relationships between the numerous 'broch-like' structures and the brochs themselves. There is undoubtedly a long tradition of drystone building in the Atlantic Province, and there may be a similarly lengthy period of promontory fortification (Lamb 1980). It is also possible to speculate on the significance of the 'broch-like' structures in Argyll as prototypes in the develop-
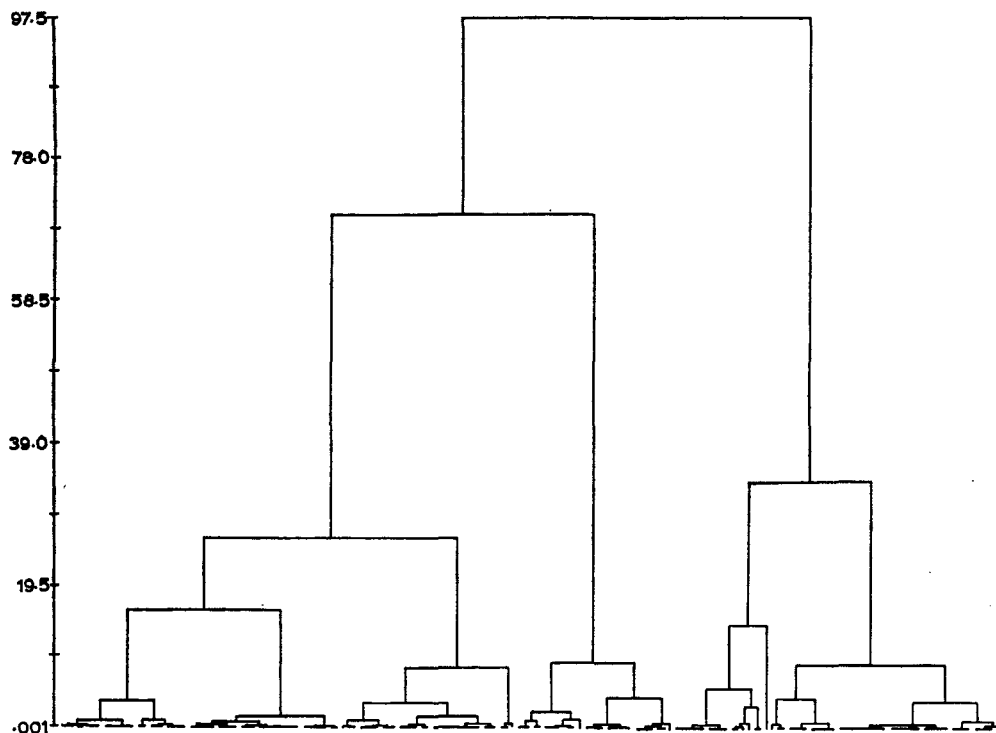
FIG 8   Dendrogram produced by Ward's method of hierarchical fusion, using the dimensions of 156 brochs and 12 'broch-like' structures

ment of ground-galleried brochs in the west. In the north, the early dates from Bu on Orkney (Hedges & Bell 1980, 90) suggest a possible prototype in the development from such structures as courtyard houses (Hamilton 1956, 18–25), round-houses (Renfrew 1979, 198) and perhaps even hut circles (Fairhurst & Taylor 1970; Fairhurst 1971), while exciting possibilities are held out by the discovery of a ring-fort beneath a broch at The Howe on Orkney (Hedges & Bell 1981). The dimensions of the Quanterness round-house are in fact only just off the scatter diagram of broch dimensions (fig 2). It is possible to speculate that the 'semi-brochs' are a variant of more standard forms of promontory fortification, using the hollow-wall technique which had been developed elsewhere.

All that is to be gained from such speculations is the construction of models which can be tested by excavation, and this is now the major requirement for the study of brochs and related sites in the Atlantic Province. As more measurements are collected from brochs, galleried duns and even duns, it is quite likely that the boundaries between them will be difficult to determine by dimensions alone. It may be possible to add circularity as a useful criterion for the classification of ring-defences, but it will probably depend ultimately on the cultural associations and the full range of architectural details (as in Graham 1948, 57 for example), where these survive, to distinguish between the various elements of the stone fort tradition recognized by Childe (1935, 204).

The fact that Graham found so little variation from circularity in the ground-plans of brochs suggests that this shape was intended by the designers. Of 30 brochs carefully measured by

Dr MacKie, most were within a mean error of 9·3 cm from a true circle (1975, 85), but again the lack of objective sampling control must qualify any conclusions. The reason for brochs apparently being distinctive in this way may be connected with a desire for maximum stability in a high, hollow wall. It has been suggested, however, that broch walls were laid out to complex geometrical shapes similar to those found in stone circles by Professor Alexander Thom, and using a standard unit of length only slightly larger than Thom's 'Megalithic Yard'. Three of MacKie's sample of 30 brochs are described as being non-circular: two are ellipses (Dun Borodale and Dun Torcuill) and one (Ness) follows the outline of Thom's type A flattened circle (*ibid*, figs 5.4 & 5.5).

One of the elliptical sites was measured at between 1·5 and 2 m above the buried ground level, and only two short stretches of walling, sometimes leaning inwards, were visible. The Royal Commission recorded this site, Dun Borodale on Raasay, as a 'broch-like structure' (RCAMS 1928, no 575), and considering that its circularity rating (the difference between the axes and their mean) is as high as 16% it might be that this site does not qualify as a true broch. It is similar in plan to Dun Kearstach (*ibid*, no 649), a larger galleried dun with a circularity rating of 22%. Excavation is obviously required to help sort out the problems of definition in this grey area, but if circularity can be regarded as a characteristic feature of brochs it is only the smaller group of 'broch-like structures' which will reveal the complex geometrical shapes.

The most complex of the non-circular sites mentioned by MacKie is Ness, in Caithness. Just under half of the inner wall has been destroyed, and the shape which 'fits' the remaining portion is made up of the arcs of no less than three circles of different radii. The short length of wall which still stands to the right of the main entrance begins to curve in sharply from the line indicated by the geometrical figure. With no evidence for the continuing line of the primary wall, examination of this site proves nothing about the construction of brochs on complex geometrical ground-plans. The other elliptical site, Dun Torcuill in North Uist, is situated on an island in Loch an Duin (*ibid*, no 172, fig 107). The restricted space available to the builders could well account for any deviations from the more usual broch ground-plan, such as the variation in wall thickness from 2·9 to 3·8 m. The Royal Commission describes the site as 'roughly circular with a diameter of 38 feet, . . . filled to a depth of about 4 feet with stones . . .' (*ibid*, 53). MacKie gives no figures for the major and minor axes of the ellipse, and presumably the measurements were not taken at ground level. No account was taken of the possible influence of topographical restrictions when any of the 30 brochs were measured (MacKie, pers comm).

The conclusions which MacKie draws from this study are that the principles of designing circular sites were either re-invented in the Iron Age, with a very close resemblance between standard units, or that the knowledge was handed down over the many generations from the Late Neolithic (MacKie 1975, 87). Thirty brochs from the surviving sample of about 500 provided evidence for a 'broch yard' of 0·839 m (compared with the Megalithic Yard of 0·829 m), and discrepancies were accounted for by invoking an 8·5 cm variation in the length of the original measuring rods. Taking this into consideration, the two units can be treated as identical, but whether they relate to actual measuring rods or to an average human pace remains to be demonstrated convincingly.

Since circularity can probably be linked to stability, it was in the builders' interest to get as close to a circle as possible, and the observations of Graham and MacKie suggest that this was achieved in the majority of cases which they examined. However, Graham also pointed out that 'the external outline . . . is liable to diverge from the circle to a greater degree than the inner . . .' (1947, 77), so it would no doubt be easy to find a multitude of complex geometrical shapes in the outline of the outside wall as well as of the courtyard. Moreover, if circular sites are more stable, they are more likely to survive in measurable form than those which depart drastically from this

shape, so a sample of brochs which are still in a measurable condition is perhaps more likely to consist of circular or near-circular sites. The underlying problem is the same for brochs as for stone circles, namely the extent to which we are guilty of reading modern technological sophistication into what may have been less exact ancient practices. The whole question has been a subject for heated debate amongst those concerned with earlier prehistory, and until more data are available in a representative sample it will remain a red herring in the Iron Age.

The other information included as binary data in the analysis, without affecting the actual computation of similarities between the numeric variables, concerns the structural classification of the sites as solid-based, ground-galleried, transitional or unknown (MacKie 1965, 101). Only slightly more than half of the sites in each cluster are of identifiable type, so few conclusions can be drawn from this part of the analysis with any certainty (fig 6c). Solid-based brochs are most common overall, a fact which may reflect the influence of differential survival and biassed sampling, and they are distributed remarkably evenly between the three clusters. Greater differences can be detected in the occurrence of the other structural types, with transitional types characterizing the Caithness/Outer Hebrides group, and ground-galleried types predominant in the Skye/Orkney group. Neither of these types is strongly represented in the Shetland/Sutherland group, which may be more important for the definition of this group than the recorded presence of 14 solid-based brochs.

CONCLUSIONS

It is very tempting to draw conclusions from these results which will help to elucidate the problems of the origins and development of brochs. Maintaining the traditional assumption that solid-based brochs are structurally more advanced, it could be claimed that this type has evolved fairly evenly in all three groups, while the lack of ground-galleried types in the Shetland/Sutherland group may indicate that these areas received mainly the developed form from Orkney and the West Coast. The fact that none of the clusters contain all the sites from one region could also be used as an argument for the spread of a broch design of certain dimensions as well as structural type, followed by slightly diverging evolution in different areas resulting in the three main types. This idea, reminiscent as it is of Darwin's observations on Galapagos finches (Darwin 1859, 385–6), may be simplistic when applied to the complexity of forces influencing broch design. It may not, for example, be the greatest number of brochs of a particular size in each region which is most important. Hierarchies could be based on considerations of status and display as well as function or structural stability, and brochs of unusual dimensions may have been intentionally designed to be different from the common types. While they may be interesting in respect of the data being examined at present, though, it is hardly acceptable for these speculations to be extended from such a sample to the whole population, but it is hoped that they will prove thought-provoking as more information is retrieved by excavation.

In terms of percentages from each region (fig 6c), the solid-based type is predominant in Shetland in cluster one, in Caithness in cluster two, and in Orkney in cluster three—brochs from each of these regions being the most numerous in their cluster. The structural stability which is assumed for these brochs during their primary use may also have helped to preserve them, making them more likely to attract the attention of 19th-century antiquarians. However, the large number of unclassifiable sites may contain a higher proportion of transitional or ground-galleried types, whose instability has led to their more complete collapse. Going purely by the regional percentage count, the Shetland/Sutherland group remains dominated by solid-based brochs, but

the other two groups are dominated by unknown types. Only when many more of these sites have been examined will it be possible to arrive at any significant conclusions.

The usefulness of cluster analysis in the typological study of archaeological data has been described elsewhere (Doran & Hodson 1975, ch 9; Hodson 1980). While the present analysis supports this general view, it highlights the major point that the quality of the results is only as good as the quality of the original data—the 'garbage in, garbage out' cliché of the computer age. A classification of brochs into three groups has been suggested, each of which has representatives in every part of the main Broch Province, but which can nevertheless be labelled by the predominant areas in each. The classification is based on two variables from 120 sites, so it is possible to present a 'type' based on the cluster mean and standard deviation for each variable (table 6). The structural class to which just over half the sites belong is known, but this is felt to be too unrepresentative a sample to allow any useful hypotheses to be suggested.

RECOMMENDATIONS

Specific aspects of the data-base are obviously in need of improvement to produce a tighter structural classification.

1 Perhaps the most important point is that the information must be collected from sites which have been drawn by an acceptable strategy from the total sample of surviving sites (which is itself no more than a biassed sample of the original population of all brochs which were ever built). Only then can hypotheses be based on the distribution of solid-based brochs, for example, without too great a likelihood of the results having been influenced if this type of site has a better chance of surviving in measurable form.

2 More variables are needed to add to the information provided by internal diameter and wall thickness. Any approach to grouping sites on these variables alone will not produce a robust classification which is generally applicable (Hodson 1980, 3), and indeed the dissection of relatively homogeneous samples by cluster analysis is open to debate (Everitt 1974, 74). Hypotheses have been based on such features as the distance of the door checks along the entrance passage, the height and type of scarcement ledge, and the number and position of mural cells, guard cells, and stairs (MacKie 1965, 110; Graham 1947, 54–70). Ideally, information on these features should be available for the whole sample before any conclusions are arrived at. For example, is there any correlation between the distance at which the door is placed along the entrance passage, and the thickness of the wall through which the passage runs—both of which are apparently greater in the north than the west although only 62 measurements are available (MacKie 1973, figs 10 & 11). This may lead to a fairly simple explanation for something which has been claimed to represent increasing sophistication in broch design, but it could also add another detail to the overall classification of the sites.

3 This analysis has attempted a typology of brochs, without differentiating between sites which can be rigorously defined as such and sites which may only be 'probables'. The criterion of evidence for the existence of upper galleries (MacKie 1965, 103) may introduce bias if applied strictly. With the collapse of a broch wall, upper galleries are more likely to survive the nearer they are to ground level. In a solid-based broch, even the first gallery would have been higher than that of a ground-galleried broch, so this vital diagnostic feature is less likely to survive when the walls fall into ruin. In such circumstances, sites are usually classified on the basis of their ground plans (Main 1979, 4), but this may prove inadequate in some cases and also evades the height issue. What may have been the most important structural variable to the original builders may not even be recoverable from circumstantial evidence in the present day, and a classification

of sites by surviving height is largely irrelevant (Scott 1947, 10). Future work may, however, refine the classification of broch-like structures and it will be interesting to see how any changes stand in relation to the groups suggested above.

4 The evidence recovered by excavation will provide not only more information to improve the classification of brochs, but also material which can be used as an important check on the results of typological analyses. A considerable amount of excavation may in fact be necessary before a properly sampled data-base can be established, and with present organization and financing this will inevitably take some time. Of most importance will be evidence for the relative dating of brochs, so that hypotheses on seriation can be added to the results of purely physical classifications. At present there exists a theory of west to north development based mainly on assumed structural evolution (MacKie 1965, 127–29), and north to west development based on the distribution of successive quern types (Caulfield 1980). The dating evidence consists at present of two dates for the construction of Dun Mor Vaul on Tiree, of 1195 ± 90 bc (GaK 1096) and ad 60 ± 90 (GaK 1097) (MacKie 1974a, 92), the first date being assumed to have been obtained from peat charcoal. At Bu on Orkney, dates of 490 ± 65 (GU 1152) and 510 ± 80 bc (GU 1154) were obtained from animal bone from the occupation phase of the site. The end of the occupation of the broch at Leckie, a Central Mainland site, has been dated to ad 45 ± 110 (GX 2779), but the length of occupation is not known (MacKie 1974b, 8). C14 dates from unspecified sources and contexts date the construction of the broch of Buchlyvie, also in Stirlingshire, to the second half of the 1st century ad (Main 1979, 10). The picture is therefore very unclear at the moment; the early dates from Bu make it earlier than the supposed 'broch prototypes' in the west (MacKie 1971, 42–3) by almost five centuries, and it is also a solid-based type which has been suggested as being more structurally advanced, and therefore later, than the ground-galleried types of the Caithness/Outer Hebrides and Skye/Orkney groups. The improvement of dating will be a major factor in dispelling much of the uncertainty which results from the present lack of information concerning brochs and related monuments.

The most obvious conclusion of this typological analysis of broch structure is that more data are needed in order to present a sound and relevant classification, but it is hoped that the qualified conclusions and liberal speculations presented in this paper will provide useful points for future discussion, and that the method of numerical taxonomy used will continue to prove useful as more data are discovered. The ball is now left firmly in the court of those who are able to provide this information by excavation and fieldwork.

ACKNOWLEDGMENTS

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